PART – A

Answer **all** questions. **Each** question carries **1** mark.

1. Define characteristic function of a random variable.

2. Show that $V(ax + b) = a^2 V(x)$.

3. If X and Y are two independent random variables, show that cov $(x, y) = 0$.

4. Define gamma distribution with parameters m and p.

5. If $X \sim N(5, 2)$ and $Y \sim N(7, 3)$ and if X and Y are independent, obtain the distribution of $x + y$.

6. Give the relationship between Beta distribution of the first kind and second kind.

$(6 \times 1 = 6)$

PART – B

Answer **any six** questions. **Each** question carries **2** marks.

7. State and prove the addition theorem of mathematical expectation.

8. Define joint probability density function of continuous random variables (x, y) and give its important properties.

P.T.O.

9. The joint pdf of X and Y is

$$f(x,y) = \begin{cases} x+y & ; \quad 0 < x < 2, 0 < y < 1 \\ 0 & ; \quad \text{elsewhere} \end{cases}$$

   Show that X and Y are stochastically independent.

10. Explain the lack of memory property of the geometric distribution.

11. Write down the important properties of the normal distribution.

12. X is normally distributed with mean 30 and S.D. 5 compute $P(1x - 5 | \leq 20)$.

13. Obtain the m.g.f. of the exponential distribution and hence obtain its mean and variance.

14. State Bernoullis' law of large numbers.   (6×2=12)

PART – C

Answer **any four** questions. **Each** question carries **3** marks.

15. Define moment generating function (m.g.f.) of a random variable X. Explain how to obtain moments of X from the m.g.f.

16. If $f(x) = 30x^4 (1 - x)$, $0 \leq x \leq 1$, find E(x) and V(x).

17. Obtain the mode of the Binomial distribution.

18. Heights of 1000 students are found to be normally distributed with mean 66 inches and S.D. 5 inches. Find the number of students with heights.
    i) between 65 and 70 inches   ii) More than 72 inches

19. A random variable X is uniformly distributed over $[a,b]$. If $E(x) = \dfrac{1}{2}$ and $V(x) = \dfrac{3}{4}$, find the values of a and b.

20. Examine whether the weak law of large numbers holds good for the sequence $\{X_n\}$ of independent random variables where

$$P\left(X_n = \frac{1}{\sqrt{n}}\right) = \frac{2}{3}, \quad P\left(X_n = \frac{-1}{\sqrt{n}}\right) = \frac{1}{3}.$$ (4×3=12)

PART – D

Answer **any two** questions. **Each** question carries **5** marks.

21. The m.g.f. of a random variable X is of the form $M_x(t) = (0.4\, e^t + 0.6)^8$. What is the m.g.f. of $Y = 3x + 2$? Evaluate E(x) and V(x) ?

22. The joint probability function of a bivariate discrete random variable is given by

    $f(x_1, x_2) = C (2x_1 + x_2)$ where

    $(x_1, x_2) = (1, 1), (1, 2), (2, 1), (2, 2), (1, 3)$ and $(2, 3)$

    i) Find C

    ii) Compute $E(X_1 | X_2 = 2)$.

23. Derive the mean deviation about mean of the normal distribution.

24. State and prove Chebychev's inequality.   (5×2=10)

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