23. The joint probability density function of a two dimensional random variable (x, y)

is given by $f(x, y) = \begin{cases} 2 & 0<x<1, \quad 0<y<x \\ 0 & \text{otherwise} \end{cases}$

i) Find the marginal density function of x and y.

ii) Find the conditional density function of x gives y and conditional density function of y gives x.

24. What do you mean by marginal and conditional distributions. The following table represents the joint probability distribution of a discrete random variable (x, y)

y \ x	1	2	3
1	$\frac{1}{12}$	$\frac{1}{6}$	0
2	0	$\frac{1}{9}$	$\frac{1}{5}$
3	$\frac{1}{18}$	$\frac{1}{4}$	$\frac{2}{15}$

i) Evaluate marginal distribution of x.

ii) Evaluate the conditional distribution of y give x = 2.

Reg. No. : ....

Name : ....

II Semester B.Sc. Degree (CCSS – 2014 Admn. – Regular)
Examination, May 2015
COMPLEMENTARY COURSE IN STATISTICS
(For Maths/Comp. Sci./Electronics Core)
2C02 STA : Probability Theory and Random Variables

Time : 3 Hours Max. Marks : 40

PART – A

Answer **all** the **6** questions. **(6×1=6)**

1. Define random experiment. Give an example.
2. Define classical definition of probability.
3. Define probability space.
4. State multiplication theorem on probability.
5. Define random variable.
6. Define conditional distribution.

PART – B

Answer **any 6** questions. **(6×2=12)**

7. State the axioms of probability.
8. Define distribution function and state its properties.
9. Two six-faced unbiased dice are thrown. Find the probability that the sum of the numbers shown is 7 or their product is 12.

P.T.O.

10. If $f_1(x)$ and $f_2(x)$ are p.d.f's and $\theta_1 + \theta_2 = 1; 0 < \theta_1, \theta_2 < 1$ examine whether $g(x) = \theta_1 f_1(x) + \theta_2 f_2(x)$ is a p.d.f.

11. For the following density function $f(x) = C\ x^2(1-x),\ 0 < x < 1$. Find the value of C.

12. Given the probability function

x	0	1	2	3
P(x)	0.1	0.3	0.5	0.1

Find the probability function of $y = x^2 + 2x$.

13. From a lot of 10 items containing 3 defectives, a sample of 4 items is drawn at random. Find the probability distribution of the no. of defective items selected.

14. Two discrete random variables X and Y have

$P(X=0\ \ Y=0) = \frac{2}{9}$ $P(X=0\ \ Y=1) = \frac{1}{9}$

$P(X=1\ \ Y=0) = \frac{1}{9}$ $P(X=1\ \ Y=1) = \frac{5}{9}$

Examine whether X and Y are independent.

PART – C

Answer **any 4** questions. **(4x3=12)**

15. State and prove Bayes theorem.

16. A problem in statistics is given to three students A, B and C whose chances of solving it are $\frac{1}{2}, \frac{3}{4}$ and $\frac{1}{4}$ respectively. What is the probability that the problem will be solved if all of them try independently ?

17. Let X be a continuous random variable with p.d.f. $f(x) \begin{cases} ax & 0 \le x \le 1 \\ a & 1 \le x \le 2 \\ -ax+3a & 2 \le x \le 3 \\ 0 & \text{elsewhere} \end{cases}$

i) Determine the constant a.

ii) Compute $P(X \le 1.5)$

18. Find K so that $f(x, y) = k\,x\,y,\ \ 1 \le x \le y \le 2$ will be a probability function.

19. Let $f(x) = \begin{cases} \frac{1}{2} & -1 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$ be the p.d.f. of a r.v.X. Find the p.d.f. of $y = x^2$.

20. The joint p.d.f. of a two dimensional random variable (X, Y) is given by

$$f(x, y) = \begin{cases} \frac{x^3y^3}{16} & 0 \le x \le 2,\ 0 \le y \le 2 \\ 0 & \text{elsewhere} \end{cases}$$

Find the marginal densities of X and Y. Also find the distribution function of X and Y.

PART – D

Answer **any 2** questions. **(2x5=10)**

21. The odds against student X solving a problem are 8 to 6 and odds in favour of student Y solving the problem are 14 to 16.
    1) What is the probability that the problem will be solved if they both try independently of each other ?
    2) What is the probability that none of them is able to solve the problem.

22. A random variable X has the following probability distribution :

Value of X **x**	0	1	2	3	4	5	6	7	8
p(x)	a	3a	5a	7a	9a	11a	13a	15a	17a

1) Determine the value of a
2) Find $P(x < 3)$, $P(x \ge 3)$ $P(0 < x < 5)$
3) What is the smallest value of x for which $P(X \le x) > 0.5$?
4) Find out the distribution function of X.