



Reg. No. : .....

Name : .....



K20U 0326

II Semester B.Sc. Degree (CBCSS – Supplementary/Improvement)  
Examination, April 2020  
(2014-2018 Admissions)  
CORE COURSE IN STATISTICS  
2B02 STA : Probability Theory

Time : 3 Hours

Max. Marks : 48

*Instruction : Use of Calculators and Statistical tables are Permitted.*

PART – A  
(Short Answer)

Answer all the questions.

(6×1=6)

1. Define a random experiment and give an example.
2. State the addition theorem of probability.
3. Define the probability density function of a continuous random variable.
4. When do you say that two random variables are independent ?
5. What is meant by correlation between two variables ?
6. Define characteristic function of a random variable.

PART – B  
(Short Essay)

Answer any seven questions.

(7×2=14)

7. Define (i) mutually exclusive and (ii) equally likely events. Give one example for each of them.
8. Given  $P(A) = \frac{1}{3}$ ,  $P(B) = \frac{1}{4}$  and A and B are independent events. Find  $P(A \cap B)$  and  $P\left(\frac{A}{B}\right)$ .
9. Write the axiomatic definition of probability.

P.T.O.



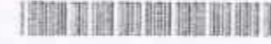
10. Distinguish between discrete and continuous random variables. Illustrate with examples.
11. If  $X$  has the density function  $f(x) = kx$ ,  $0 \leq x \leq 2$ , find  $k$ . What is the distribution function of  $X$ ?
12. Find the expectation of  $X$ , if  $X \sim f(x)$ , where  $f(x) = \frac{1}{4}(3x - 1)$ ,  $0 \leq x \leq 2$ .
13. Distinguish between raw and central moments.
14. Prove that  $E(X) = E[E(X/Y)]$ .
15. Find the probability generating function of a random variable  $X$  with probability mass function  $P(X = k) = \frac{e^{-\lambda} \lambda^k}{k!}$ ,  $k = 0, 1, 2, \dots$

PART - C  
(Essay)

Answer any four questions.

(4x4=16)

16. Explain pair wise and mutual independence of events. Also write the mutual implications if any.
17. Find the constant  $k$  such that the function
- |        |      |           |        |       |
|--------|------|-----------|--------|-------|
| $x$    | 1    | 2         | 3      | 4     |
| $f(x)$ | $2k$ | $k^2 + k$ | $2k^2$ | $k^2$ |
- is a probability mass function. Also find  $P(X \leq 2)$ .
18. If  $X$  has the density function  $f(x) = 1$ ,  $0 < x < 1$ , find the distribution of  $e^x$ .
19. Show that  $\text{Var}(aX + bY) = a^2 \text{var}(X) + b^2 \text{var}(Y)$  if  $X$  and  $Y$  are independent.
20. If  $f(x, y) = x + y$ ,  $0 \leq x \leq 1$ ,  $0 \leq y \leq 1$ , find (a) the conditional density of  $X$  given  $Y$  and (b)  $E(X/Y)$ .
21. Show that the cumulant generating function is used for generating cumulants.



PART - D  
(Long Essay)

Answer any two questions.

(2x6=12)

22. i) State and prove Bayes theorem.
- ii) In a factory machines I, II, III are all producing springs of the same length. Of their production machines I, II, III produce 2%, 1% and 3% defectives respectively. Of the total production of springs in the factory, machine I produces 35%, machine II produces 25% and machine III produces 40%. If one spring is selected at random from the total springs produced in a day, find the probability that the spring is taken from machine III given it is defective.
23. The joint density function of a bivariate random variable is
- $$f(x, y) = \frac{xy^2}{30} \text{ when } x = 1, 2, 3 \text{ and } y = 1, 2.$$
- Examine whether  $X$  and  $Y$  are independent.
24. i) Establish the relationship between raw and central moments.
- ii) If  $\mu_1' = 1$ ,  $\mu_2' = 3$ ,  $\mu_3' = 5$ ,  $\mu_4' = 6$ , find  $\mu_1$ ,  $\mu_2$ ,  $\mu_3$  and  $\mu_4$ .
25. i) Prove that the moment generating function (m.g.f) of two independent random variables is the product of their m.g.f.s.
- ii) Find the m.g.f of  $X$  where  $X$  has the density  $f(x) = \theta e^{-\theta x}$ ,  $0 < x < \infty$ . Hence find its mean and variance.