K20U 0326

Reg. No. : ....

Name : ....

II Semester B.Sc. Degree (CBCSS – Supplementary/Improvement) Examination, April 2020
(2014-2018 Admissions)
CORE COURSE IN STATISTICS
2B02 STA : Probability Theory

Time : 3 Hours | Max. Marks : 48

Instruction : *Use of Calculators and Statistical tables are **Permitted**.*

PART – A
(Short Answer)

Answer **all** the questions. (6×1=6)

1. Define a random experiment and give an example.
2. State the addition theorem of probability.
3. Define the probability density function of a continuous random variable.
4. When do you say that two random variables are independent ?
5. What is meant by correlation between two variables ?
6. Define characteristic function of a random variable.

PART – B
(Short Essay)

Answer **any seven** questions. (7×2=14)

7. Define (i) mutually exclusive and (ii) equally likely events. Give one example for each of them.
8. Given $P(A) = \frac{1}{3}, P(B) = \frac{1}{4}$ and A and B are independent events. Find $P(A \cap B)$ and $P\left(A/B\right)$.
9. Write the axiomatic definition of probability.

P.T.O.

10. Distinguish between discrete and continuous random variables. Illustrate with examples.

11. If X has the density function $f(x) = kx,\ 0 \le x \le 2$, find k. What is the distribution function of X ?

12. Find the expectation of X, if $X \sim f(x)$, where $f(x) = \frac{1}{4}(3x - 1),\ 0 \le x \le 2$.

13. Distinguish between raw and central moments.

14. Prove that $E(X) = E[E(X/Y)]$.

15. Find the probability generating function of a random variable X with probability mass function $P(X = k) = \frac{e^{-\lambda}\lambda^k}{k!},\ k = 0, 1, 2, \ldots$

PART – C

(Essay)

Answer **any four** questions. **(4x4=16)**

16. Explain pair wise and mutual independence of events. Also write the mutual implications if any.

17. Find the constant k such that the function

x	1	2	3	4
f(x)	2k	$k^2 + k$	$2k^2$	k^2

is a probability mass function. Also find $P(X \le 2)$.

18. If X has the density function $f(x) = 1,\ 0 < x < 1$, find the distribution of e^X.

19. Show that $Var(aX + bY) = a^2 \, var(X) + b^2 \, var(Y)$ if X and Y are independent.

20. If $f(x, y) = x + y,\ 0 \le x \le 1,\ 0 \le y \le 1$, find (a) the conditional density of X given Y and (b) E (X/Y).

21. Show that the cumulant generating function is used for generating cumulants.

PART – D

(Long Essay)

Answer **any two** questions. (2x6=12)

22. i) State and prove Bayes theorem.

    ii) In a factory machines I, II, III are all producing springs of the same length. Of their production machines I, II, III produce 2%, 1% and 3% defectives respectively. Of the total production of springs in the factory, machine I produces 35%, machine II produces 25% and machine III produces 40%. If one spring is selected at random from the total springs produced in a day, find the probability that the spring is taken from machine III given it is defective.

23. The joint density function of a bivariate random variable is

$$f(x, y) = \frac{xy^2}{30} \text{ when } x = 1, 2, 3 \text{ and } y = 1, 2.$$

Examine whether X and Y are independent.

24. i) Establish the relationship between raw and central moments.

    ii) If $\mu_1' = 1, \mu_2' = 3, \mu_3' = 5, \mu_4' = 6$, find μ_1, μ_2, μ_3 and μ_4.

25. i) Prove that the moment generating function (m.g.f) of two independent random variables is the product of their m.g.f,s.

    ii) Find the m.g.f of X where X has the density $f(x) = \theta e^{-\theta x},\ 0 < x < \infty$. Hence find its mean and variance.

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