24. If X and Y are two random variables having joint p.d.f.



K18U 0517

Il Semester B.Sc. Degree (C.B.C.S.S. - Reg./Supple./Imp.)

Examination, May 2018 CORE COURSE IN STATISTICS 2B02 STA: Probability Theory (2014 Admn. Onwards)

Time: 3 Hours

Max. Marks: 48

PART-A

Short Answer. Answer all the 6 questions.

- 1. Define conditional probability.
- 2. Distinguish between discrete and continuous random variables.
- 3. Define distribution function.
- 4. State multiplication theorem on expectation.
- 5. State the relation between second central moment and first and second raw
- 6. Define cumulant generating function.

Short Essay. Answer any 7 questions. (7x2=14)

- 7. If A and B are two events such that $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$ and $P(AB) = \frac{1}{8}$. Find P(A|B) and P(A|BC)
- 8. Explain the concept of statistical regularity.
- 9. Two random variables X and Y have the joint p.d.f.

$$f(x, y) = k(x^2 + y^2), 0 < x < 2, 1 < y < 4$$

= 0, elsewhere. Find k.

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10. If X is a continuous random variable with p.d.f.

$$f(X) = 2x, 0 < x < 1$$

and Y = 3X + 1, then find the p.d.f. of Y.

- 11. Define conditional expectation. Prove that E[E(X|Y)] = E(X).
- Give an example to show that pairwise independence does not imply mutual independence.
- 13. The joint p.d.f. of two random variables (X, Y) is given by

$$f(x, y) = 2, 0 < x < y < 1$$

= 0, elsewhere.

Find the conditional mean of X given Y = y.

- 14. State any two drawbacks of moment generating function.
- 15. Define probability generating function.

Essay. Answer any 4 questions.

 $(4 \times 4 = 16)$

- 16. From a vessel containing 3 white and 5 black balls, 4 balls are transferred into an empty vessel. From this vessel a ball is drawn and is found to be white. What is the probability that out of four balls transferred, 3 are white and 1 black.
- 17. X and Y are two random variables having the joint density function, $f(x, y) = \frac{1}{27} (2x + y), \text{ where } x \text{ and } y \text{ can assume only the integer values 0, 1 and } 2. Find the conditional distribution of Y for X = x.}$
- 18. State and prove Baye's theorem.
- 19. State and prove any two properties of characteristic function.
- 20. If X and Y have the joint p.d.f. as follows:

$$(x, y) : (0, 0) (0, 1) (0, 2) (1, 0) (1, 1) (1, 2)$$

Find Cov (X, Y).

21. Find k so that f(x, y) = kx (y - x), $0 \le x \le 4$, $4 \le y \le 8$ will be a bivariate probability density function.



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PART-D

Long Essay. Answer any 2 questions.

(2×6=12)

- 22. a) Distinguish between frequency and axiomatic approaches to probability.
 - b) If two dice are thrown, what is the probability that the sum is :
 - i) greater than 8 and
 - ii) neither 7 nor 11?

23. If
$$f(x, y) = k(x + y + xy)$$
, $0 < x < 1$, $0 < y < 1$.

= 0, otherwise

Determine k, the marginal densities and the conditional density of X | Y.

. 24. If X and Y are two random variables having joint p.d.f.

$$f(x, y) = 2 - x - y, 0 < x < 1; 0 < y < 1$$

= 0, elsewhere

. Find correlation coefficient ρ_{xy} .

25. Define conditional variance. Show that variance of a random variable can be written as the sum of the expectation of the conditional variance and the variance of the conditional expectation.