Reg. No. : ....

Name : ....

Second Semester B.Sc. (Hon's) Mathematics Degree (CBCSS – OBE – Supplementary/Improvement) Examination, April 2025
(2021 to 2023 Admissions)
Core Course
2B05BMH : CALCULUS – II

Time : 3 Hours | Max. Marks : 60

SECTION – A

Answer **any 4** questions. **Each** question carries **1** mark. **(4×1=4)**

1. Define Cycloid.
2. Write the standard equation of the "four-leaved rose".
3. Define sequence of real numbers.
4. State alternating series estimation theorem.
5. Write down the general equation of the hyperboloid of one sheet.

SECTION – B

Answer **any 6** out of 9 questions. **Each** question carries **2** marks. **(6×2=12)**

6. Eliminate parameter to find a Cartesian equation of the curve $x = 3 - 4t$, $y = 2 - 3t$.
7. Find the points on the curve $x = t^3 - 3t$, $y = t^2 - 3$, where the tangent is horizontal or vertical.
8. Test the series $\sum_{n=1}^{\infty} \frac{1}{2^n - 1}$ for convergence or divergence.
9. Show that every absolutely convergent series is convergent.

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10. Find the interval of convergence of the series $\sum_{n=1}^{\infty} \frac{(x-3)^n}{n}$.
11. Find the angle between the planes $x + y + z = 1$ and $x - 2y + 3z = 1$.
12. Prove that $\frac{d}{dt}[\vec{u}(t)\cdot\vec{v}(t)] = \vec{u}'(t)\cdot\vec{v}(t) + \vec{u}(t)\cdot\vec{v}'(t)$.
13. Find the directional derivative of $f(x, y) = x^3y^4 + x^4y^3$ at the point (1, 1) in the direction given by the angle $\theta = \frac{\pi}{6}$.
14. Find y' if $x^3 + y^3 = 6xy$.

SECTION – C

Answer **any 8** out of 12 questions. **Each** question carries **4** marks. **(8×4=32)**

15. Show that the surface area of a sphere of radius r is $4\pi r^2$.
16. For $\alpha \in \mathbb{Q}$, calculate $\lim_{n\to\infty} \sin(n!\alpha\pi)$.
17. Determine whether the series $\sum_{n=3}^{\infty} \frac{n+2}{(n+1)^3}$ converges or diverges ? Justify your answer.
18. Determine whether the series $\sum_{n=1}^{\infty} \frac{(-1)^n e^{\frac{1}{n}}}{n^3}$ is absolutely convergent, conditionally convergent or divergent. Justify your answer.
19. Find the Taylor series of $f(x) = \sin x$ centered at $\frac{\pi}{3}$.
20. Find symmetric equations for the line of intersection L of the planes $5x - 2y - 2z = 1$ and $4x + y + z = 6$.
21. Find the parametric equations for the tangent line to the helix with vector equation $r(\vec{t}) = 2\cos t\,i + \sin t\,j + tk$ at the point $\left(0, 1, \frac{\pi}{2}\right)$.
22. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if $x^3 + y^3 + z^3 + 6xyz = 1$.

23. Find directional derivative of $f(x, y, z) = x \sin yz$ at (1, 3, 0) in the direction of $\vec{v} = i + 2j - k$.
24. A particle moves with position function $r(\vec{t}) = t^2 i + t^2 j + t^3 k$. Find the tangential and normal components of acceleration.
25. Determine the set of points at which the function
$$f(x, y) = \begin{cases} \frac{xy}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$
is continuous.
26. Find the local maximum and minimum values and saddle points of $f(x, y) = x^4 + y^4 - 4xy + 1$.

SECTION – D

Answer **any 2** out of 4 questions. **Each** question carries **6** marks. **(2×6=12)**

27. a) Show that if $a_n > 0$ and $\lim_{n\to\infty} na_n \neq 0$, then $\sum a_n$ is divergent.
    b) Find the radius of convergence and interval of convergence of the series $\sum_{n=0}^{\infty} \frac{n(x+2)^n}{3^{n+1}}$.
28. a) Find the extreme values of $f(x, y) = x^2 + 2y^2$ on the disk $x^2 + y^2 \leq 1$.
    b) Use Lagrange multipliers to prove that the rectangle with maximum area that has a given perimeter P is a square.
29. a) Find the arc length of the circular helix with vector equation $r(\vec{t}) = \cos t\,i + \sin t\,j + tk$ from the point (1, 0, 0) to the point (1, 0, 2π).
    b) Find an equation of the plane that passes through the points (0, – 2, 5) and (– 1, 3, 1) and is perpendicular to the plane $2z = 5x + 4y$.
30. a) Find the points on the sphere $x^2 + y^2 + z^2 = 4$ that are closest to and farthest from the point (3, 1, – 1).
    b) Find the area of the region enclosed by one loop of the curve $r^2 = \sin 2\theta$.

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