Reg. No. : ....

Name : ....

Second Semester B.Sc. (Hon's) Mathematics Degree (C.B.C.S.S. – Supplementary) Examination, April 2025
(2019 and 2020 Admissions)
BHM 202 : ABSTRACT ALGEBRA AND LINEAR ALGEBRA

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any 4** questions out of 5 questions. **Each** question carries **1** mark.

1. Give a subgroup of $\mathbb{Z}$.
2. Write an example of abelian group, which is not cyclic.
3. Give a basis of $\mathbb{R}^2$ over $\mathbb{R}$.
4. What is the dimension of $\mathbb{R}^n$ over $\mathbb{R}$?
5. Is $T:\mathbb{R}\to\mathbb{R}$ defined by $T(x) = x^2$ linear ? Justify your answer. **(4×1=4)**

SECTION – B

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks.

6. Define finite and infinite groups with examples.
7. Show that in a group G with binary operation $*$, there is one and only one element $e \in G$ such that $e * x = x * e = x$.
8. Is the set $G = \mathbb{Q}^c \cup \{0\}$ group under addition ?
9. Find (1, 3, 6) (2, 8) (4, 7, 5).
10. Give an example to show that the group S_3 is non-abelian.
11. Show that the set {(1, 0), (0, 1)} spanns $\mathbb{R}^2$.
12. Write all subspaces of $\mathbb{R}^3$ over $\mathbb{R}$.

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13. Find the dimension of the subspace $W = \{(x, y) \in \mathbb{R}^2 : x + y = 0\}$.
14. Find the nullity and rank of the linear transformation $T:\mathbb{R}^2 \to \mathbb{R}^2$ by $T(x, y) = (x + y, 0)$. **(6×2=12)**

SECTION – C

Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks.

15. Show that $U = \{1, -1, i, -i\}$, $i = \sqrt{-1}$ is an abelian group under multiplication.
16. Let G be a group and a be one fixed element of G. Show that $H_a = \{x \in G : xa = ax\}$ is subgroup of G.
17. Show that every cyclic group is abelian.
18. Find all cyclic subgroups of S_3.
19. Write the permutation (12345) in product of 2-cycles.
20. Let $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 3 & 5 & 4 & 6 \end{pmatrix}$ and $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 1 & 2 & 4 & 3 & 5 \end{pmatrix}$. Compute α^{-1} and β^{-1}.
21. Let $S = \{v_1, v_2, \dots, v_n\}$ is a basis for a vector space V. Then show that every element $v \in V$ can be expressed in uniquely in the form of $v = c_1v_1 + c_2v_2 + \dots + c_nv_n$, c_i are scalars for every i.
22. Define linearly independent and spanning set. Show that the set {(1, 0), (0, 1)} is linearly independent and spans $\mathbb{R}^2$.
23. Write a basis for $\mathbb{R}^3$ over $\mathbb{R}$. Express (1, 0, 0) as the linear combination of the basis.
24. Let $T:\mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x, y) = (x, 0)$. Find the matrix of T with respect to the standard basis of $\mathbb{R}^2$.
25. T and U be the linear operators on $\mathbb{R}^2$ defined by $T(x, y) = (y, x)$ and $U(x, y) = (x, 0)$. Find ToU and UoT.
26. Define null space of a linear transformation. Find the null space of the linear map $T(x, y, z) = (x - y, x + 2y, y + 3z)$. **(8×4=32)**

SECTION – D

Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks.

27. Show that in a group G, the identity element and inverse of each element is unique.
28. Write the group representation table of all groups of order 4.
29. Let V and W be finite dimensional vector spaces and $T : V \to W$ be a linear transformation. Show that $\dim(V) = \dim \mathrm{Im}(T) + \dim \mathrm{Ker}(T) = \mathrm{rank}\, T + \mathrm{Nullity}\, T$.
30. Let V be a finite dimensional vector space. Let $T : V \to V$ be a linear map. Show that the following statements are equivalent.
    i) T is bijective
    ii) Ker T = {0}
    iii) Im T = V. **(2×6=12)**

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