K25U 1453

Reg. No. : ....

Name : ....

Second Semester B.Sc. (Hon's) Mathematics Degree (CBCSS – Supplementary) Examination, April 2025 (2019 and 2020 Admissions)

BHM 204 : THEORY OF NUMBERS AND EQUATIONS

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any 4** questions out of 5 questions. **Each** question carries **1** mark. **(4×1=4)**

1. For integers a, b prove that if $a|b$ and $b|a$, then $a = \pm b$.
2. Define the greatest common divisor of two integers a and b.
3. Determine whether the Diophantine equation $14x + 35y = 93$ is solvable ?
4. For any choice of positive integers a and b; $\text{lcm}(a, b) = ab$ if and only if $\gcd(a, b) = 1$.
5. State Descartes' rule of signs for negative roots.

SECTION – B

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks. **(6×2=12)**

6. If $a|bc$, with $\gcd(a, b) = 1$, then prove that $a|c$.
7. Prove that a and b are relatively prime if and only if there exist integers x and y such that $ax + by = 1$.
8. If $a|c$ and $b|c$, with $\gcd(a, b) = 1$, then $ab|c$.
9. Use the Euclidean algorithm to obtain integers x and y satisfying $\gcd(24, 138) = 24x + 138y$.
10. Determine all solutions of $56x + 72y = 40$.

P.T.O.

11. If p is a prime, then $a^p \equiv a \pmod p$ for any integer.
12. Remove the fractional coefficients from the equation $x^3 + \frac{1}{4}x^2 - \frac{1}{16}x + \frac{1}{72} = 0$.
13. Solve the equation $x^4 + 2x^3 - 5x^2 + 6x + 2 = 0$ of which one root is $1 + \sqrt{-1}$.
14. If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, form the equation whose roots are $\alpha\beta, \beta\gamma, \gamma\alpha$.

SECTION – C

Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks. **(8×4=32)**

15. Solve the linear Diophantine equation $221x + 35y = 11$.
16. Prove that $c \equiv d \pmod n$ if and only if c and d leave the same non-negative remainder when divided by n.
17. Solve the system of three congruences $x \equiv 1 \pmod 3$, $x \equiv 2 \pmod 5$, $x \equiv 3 \pmod 7$.
18. If $ka \equiv kb \pmod n$, then $a \equiv b\left(\bmod \frac{n}{d}\right)$ where $d = \gcd(k, n)$.
19. If $p, q_1, q_2, \ldots q_n$ are all primes and $p|q_1q_2q_3\ldots q_n$, then $p = q_k$ for some k, $1 \le k \le n$.
20. Find the condition that the roots of the equation $ax^3 + 3bx^2 + 3cx + d = 0$ may be in geometric progression.
21. If the sum of two roots of the equation $x^4 + px^3 + qx^2 + rx + s = 0$ equals the sum of the other two, prove that $p^3 + 8r = 4pq$.
22. Solve the equation $6x^6 - 35x^5 + 56x^4 - 56x^2 + 35x - 6 = 0$.
23. Find the number of real roots of the equation $x^3 + 18x - 6 = 0$.
24. If α, β, γ be the roots of the equation $x^3 + px^2 + qx + r = 0$, form an equation whose roots are $\alpha - \frac{1}{\beta\gamma}, \beta - \frac{1}{\gamma\alpha}, \gamma - \frac{1}{\alpha\beta}$.
25. Find the condition that all the roots of the equation $x^3 + px + q = 0$ may be real.
26. Find $\frac{1}{\alpha^5} + \frac{1}{\beta^5} + \frac{1}{\gamma^5}$, where α, β, γ are the roots of the equation $x^3 + 2x^2 - 3x - 1 = 0$.

SECTION – D

Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks. **(2×6=12)**

27. Given integers a and b, not both of which are zero, prove that there exist integers x and y such that $\gcd(a, b) = ax + by$.
28. State and prove Fermat's little theorem.
29. If a, b, c be the roots of the equation $x^3 + px^2 + qx + r = 0$, find the equation whose roots are $bc - a^2, ca - b^2, ab - c^2$.
30. Solve the equation $x^4 - 2x^3 + 4x^2 + 6x - 21 = 0$, given that two of its roots are equal in magnitude and opposite in sign.
