K25U 1407

Reg. No. : ....

Name : ....

Second Semester B.Sc. (Hon's) Mathematics Degree (C.B.C.S.S. – O.B.E. Supplementary/Improvement) Examination, April 2025
(2021 to 2023 Admissions)
Core Course
2B06 BMH : DISTRIBUTION FUNCTIONS AND COMBINATORICS

Time : 3 Hours   Max. Marks : 60

PART – A

Answer **any four** questions from this Part. **Each** question carries **1** mark. **(4×1=4)**

1. What is the coefficient of variation of Poisson distribution with parameter λ ?
2. What are the points of inflexion of normal curve ?
3. What is the characteristic function of binomial distribution ?
4. Find ϕ (12300).
5. Find the generating function for the sequence 1, – 1, 1, – 1, 1, – 1, ...

PART – B

Answer **any six** questions from this Part. **Each** question carries **2** marks. **(6×2=12)**

6. State Lindeberg-Levy form of central limit theorem.
7. Two dice are thrown n times. Let X denote the number of thrown in which the number on the first die exceeds the number on the second die. Find the distribution of X.
8. A Poisson variate is such that $P(X = 1) = 2P(X = 2)$. Find $P(X = 0)$.
9. The mean and variance of a binomial distribution are 3 and 2 respectively. Find $P(X = 0)$.
10. Define geometric distribution and find its mean.

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11. Determine the number of positive integer n, $1 \le n \le 2000$ that are not divisible by 2, 3, 5 or 7.
12. Find the number of derangements of 1, 2, 3, 4.
13. How many integer solutions are there for the equation $c_1 + c_2 + c_3 + c_4 = 20$, $0 \le c_i$ for all $1 \le i \le 4$ with c_2 and c_3 even ?
14. Find the exponential generating function for the sequence 1, 2, 2^2, 2^3, 2^4, ...

PART – C

Answer **any eight** questions from this Part. **Each** question carries **4** marks. **(8×4=32)**

15. If X follows normal distribution with mean 30 and standard deviation 5. Find
    a) $P(25 \le X \le 35)$
    b) $P(X \ge 40)$
    c) $P(|X - 30| > 50)$
    d) $P(X < 30)$.
16. X and Y are independent binomial variate with parameters (n_1, p) and (n_2, p) respectively. Find :
    a) The distribution of X + Y.
    b) Conditional distribution of X given X + Y.
17. Show that for a Poisson distribution with unit mean, the mean deviation about mean is $\frac{2}{e}$ times the standard deviation.
18. Show that for a rectangular distribution $f(x) = \frac{1}{2a}$ where $-a < x < a$, the moment generating function about the origin is $\frac{1}{at}$ sinh at. Also show that the moments of even order are given by $\mu_{2n} = \frac{a^{2n}}{2n+1}$.
19. The marks of a set of students for a certain subject are approximately normally distributed with mean 62 and standard deviation 3. If 4 students are randomly chosen from the set, what is the probability that 3 of them have less than 60% marks ?

20. Find mean deviation from mean of normal distribution.
21. In how many ways can the 26 letters of the alphabet be permuted so that none of the patterns car, dog, pun or byte occurs ?
22. For the positive integers 1, 2, 3, ..., n – 1, n there are 11660 derangements, where 1, 2, 3, 4 and 5 appears in the first five positions. What is the value of n ?
23. Derive the rook polynomial for the standard 8 × 8 chessboard. Generalize your formula for the standard n × n chessboard, for $n \in Z^+$.
24. Determine the coefficient of x^8 in $\frac{1}{(x-1)(x-2)^2}$.
25. Describe the concept of integer partition and illustrate with an example. Derive the generating function for the number of partitions of a positive integer n into district summands. Also, find $P_d(6)$.
26. A ship carries 48 flags, 12 each of colors red, white, blue and black. Twelve of these flags are placed on a vertical pole in order to communicate a signal to other ships. How many of these signals use an even number blue flags and an odd number of black flags ?

PART – D

Answer **any two** questions from this Part. **Each** question carries **6** marks. **(2×6=12)**

27. Show Poisson distribution as a limiting form of binomial distribution.
28. If X_1 and X_2 are independent rectangular variate over [0, 1]. Find the distribution of (a) $X_1 | X_2$, (b) $X_1 + X_2$.
29. For finite sets A and B with $|A| = m \ge n = |B|$, prove using Inclusion-Exclusion principle that the number of functions from A into B is n^m.
30. a) Use generating functions to determine how many four-element subsets of $S = \{1, 2, ..., 15\}$ contains no consecutive integers.
    b) Find the coefficient of x^7 in $(1 + x + x^2 + x^3 + ...)^{15}$.

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