K25U 0831

Reg. No. : .....

Name : .....

IV Semester B.Sc. Degree (C.B.C.S.S. – O.B.E. – Regular/Supplementary/ Improvement) Examination, April 2025
(2019 to 2023 Admissions)
COMPLEMENTARY ELECTIVE COURSE IN MATHEMATICS
4C04 MAT-PH : Mathematics for Physics – IV

Time : 3 Hours Max. Marks : 40

PART – A

Answer **any four** questions. **Each** question carries **one** mark. **(4×1=4)**

1. Write one dimensional wave equation.
2. State fundamental theorem on superposition.
3. Define gradient field.
4. Define smooth surface.
5. State geometrical significance of trapezoidal rule.

PART – B

Answer **any seven** questions. **Each** question carries **2** marks. **(7×2=14)**

6. Verify that $u = e^{-t} \sin x$ is a solution of $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$.
7. Solve $u_{xx} - u = 0$.
8. Find the type of the PDE $u_{xx} + 4u_{yy} = 0$.
9. Evaluate $\int_C (x + y)\, dS$ where C is the straight line segment $x = t$, $y = (1 - t)$, $z = 0$ from (0, 1, 0) to (1, 0, 0).

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10. Find the flux of $F = (x - y)i + xj$ across the circle $x^2 + y^2 = 1$ in the xy plane.
11. Find the gradient of $f(x, y, z) = (x^2 + y^2 + z^2)^{-\frac{1}{2}}$.
12. Find a parametrization of the cylinder $x^2 + (y - 3)^2 = 9$, $0 \leq z \leq 5$.
13. If $r(u, v) = \cos u \cos v\, i + \cos u \sin v\, j + uk$, find $|r_u \times r_v|$.
14. State divergence theorem.
15. Evaluate $\int_0^1 \frac{dx}{1+x}$ using Trapezoidal rule taking $h = 0.25$.
16. Write the fourth order Runge Kutta formula.

PART – C

Answer **any four** questions. **Each** question carries **three** marks. **(4×3=12)**

17. Find solutions $u = u(x, y)$ of the PDE $u_{xy} = -u_x$.
18. Find the work done by the force field $F = (y - x^2)i + (z - y^2)j + (x - z^2)k$ along the curve $r(t) = ti + t^2j + t^3k$, $0 \leq t \leq 1$ from (0, 0, 0) to (1, 1, 1).
19. Show that $F = (e^x \cos y + yz)i + (xz - e^x \sin y)j + (xy + z)k$ is conservative over its natural domain and find a potential function for it.
20. Find the surface area of the hemisphere of radius a.
21. Find the curl of $F = (x^2 - z)i + xe^z j + xyk$.
22. Given $\frac{dy}{dx} = 1 + xy$, $y(0) = 1$, obtain the Taylor series for $y(x)$ and compute $y(0.1)$ correct to four decimal places.
23. Explain Euler's method.

PART – D

Answer **any two** questions. **Each** question carries **five** marks. **(2×5=10)**

24. Find the solution of the wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ satisfying $u(0, t) = 0$ and $u(L, t) = 0$ for all $t \geq 0$ and corresponding to the triangular initial deflection,
$f(x) = \begin{cases} \frac{2k}{L}x, & \text{if } 0 < x < \frac{L}{2} \\ \frac{2k}{L}(L - x), & \text{if } \frac{L}{2} < x < L \end{cases}$ and initial velocity zero.
25. State normal form of Green's theorem. Verify this theorem for the vector field $F = yi - xj$ and the region R is bounded by the circle $C : r(t) = a \cos t\, i + a \sin t\, j$, $0 \leq t \leq 2\pi$.
26. State Stoke's theorem. Verify this theorem for the vector field $F = yi - xj$ over the hemisphere $S : x^2 + y^2 + z^2 = 9$, $z \geq 0$, bounded by the circle $C : x^2 + y^2 = 9$, $z = 0$.
27. Use Runge-Kutta fourth order formula to find $y(0.2)$ and $y(0.4)$ given that $y' = \frac{y^2 - x^2}{y^2 + x^2}$, $y(0) = 1$.
