Reg No:.................
Name :...................

K25FY2465 B

Second Semester FYUGP Mathematics Examination
APRIL 2025 (2024 Admission onwards)
KU2DSCMAT116 (MULTIVARIABLE CALCULUS)
(DATE OF EXAM: 30-4-2025)

Time : 120 min

Maximum Marks : 70

Part A (Answer any 6 questions. Each carries 3 marks)

1. Find the limit $\lim_{(x,y)\to(1/27,\pi^3)} \cos\sqrt[3]{xy}$. 3

2. Find $\frac{dw}{dt}$ if $w = xy - z^2$, $x = \cos t$, $y = \sin t$, $z = t$. 3

3. Find $\frac{dw}{dt}$ as a function of t where $w = x^2 + y^2$, $x = \cos t$, $y = \sin t$. 3

4. Evaluate $\int_{\pi}^{2\pi}\int_{0}^{\pi}(\sin x + \cos y)\, dx\, dy$. 3

5. Evaluate $\int_{0}^{2}\int_{0}^{1}(4 - x - y)\, dy\, dx$. 3

6. Find the work done by the force field $F = \nabla f$, where $f(x,y,z) = \frac{1}{x^2+y^2+z^2}$ in moving an object along a smooth curve C joining $(1,0,0)$ to $(0,0,2)$ that does not pass through the origin. 3

7. State the formula to find the mass of a thin shell of density $\delta(x,y,z)$. 3

8. Write the formula to find the area of the smooth parameterized surface $\mathbf{r}(u,v) = f(u,v)\mathbf{i} + g(u,v)\mathbf{j} + h(u,v)\mathbf{k}$, $a \le u \le b$, $c \le v \le d$. 3

Part B (Answer any 4 questions. Each carries 6 marks)

9. Evaluate $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if $z = \frac{x^2+y^2}{x-y}$. 6

10. Evaluate $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if $z = \tan^{-1}\frac{x^2+y^2}{x+y}$. 6

11. Find f_{xx}, f_{yy}, f_{xy} and f_{yx} if $f(x,y) = x^2y - \cos y + y\sin x$ 6

12. Integrate $f(x,y,z) = x - 3y^2 + z$ over the line segment C joining the origin to the point $(1,1,1)$. 6

13. Evaluate $\int_C x\, ds$ where C is the parabolic curve $x = t$, $y = t^2$ from $(0,0)$ to $(2,4)$. 6

14. Find the surface area of the cone $z = \sqrt{x^2+y^2}$, $0 \le z \le 1$. 6

Part C (Answer any 2 question(s). Each carries 14 marks)

15. (a) Find the unit normal vector to the surface $3z^2 = 2x^2 + 6y^2$ at the point $(0,1,\sqrt{2})$.
    (b) Find the divergence of the vector function $[z^2, -2x^2, -y^2]$ at the point $(\sqrt{2},1,\sqrt{5})$.
    (c) Find the curl of the vector function $[x\sin^2(zy), e^y, xy^2z]$ at the point $\left(1,1,\frac{\pi}{2}\right)$.
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16. (a) Find the unit normal vector to the surface $z^2 = 2x^2 + y^2$ at the point $(\sqrt{2},1,\sqrt{5})$.
    (b) Find the directional derivative of the function $f(x,y,z) = x\sin^2(zy)$ at the point $\left(1,1,\frac{\pi}{2}\right)$ in the direction of the vector $\mathbf{a} = [-1,3,-4]$.
    (c) Find the curl of the vector function $[x\cos^2(zy), e^y, xy^2z]$ at the point $\left(1,1,\frac{\pi}{2}\right)$.
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17. (a) Find the volume of the region bounded above by the paraboloid $z = x^2 + y^2$ and below by the square $R: -1 \le x \le 1, -1 \le y \le 1$.
    (b) Evaluate $\int_{0}^{\pi}\int_{0}^{\sin x} y\, dy\, dx$.
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