K25U 0322

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Mathematics (Honours) Degree (CBCSS – OBE – Regular/Supplementary/Improvement) Examination, April 2025
(2021 and 2022 Admissions)
Core Course
6B25 BMH : TOPOLOGY

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any 4** questions out of 5 questions. **Each** question carries **1** mark.

1. Give an example of a metric on $\mathbb{C}$.
2. Let X be a metric space with metric d. When do we say that a sequence $\{x_n\}$ of points in X is convergent ?
3. Write the discrete topology of the set {1, 2}.
4. Let X be the subspace [–1, 1] of the real line. Determine whether the sets [–1, 0] and (0, 1] form a separation of X. Justify.
5. Give an example of a subspace of a Lindelof space, which is not Lindelof. **(4×1=4)**

SECTION – B

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks.

6. Prove that in any metric space X, each open sphere is an open set.
7. Let A be a subset of an arbitrary metric space X. Give any two properties of Int(A).
8. State Cantor's intersection theorem.
9. Let X be a set and $\mathcal{B}$ be a basis for a topology Γ on X. Then prove that Γ equals the collection of all unions of elements of $\mathcal{B}$.

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10. Prove that every finite point set in a Hausdorff space X is closed.
11. If the sets C and D form a separation of X and if Y is a connected subspace of X, then prove that Y lies entirely within either C or D.
12. Let f : X → Y be a bijective continuous function. If X is compact and Y is Hausdorff, then prove that f is a homeomorphism.
13. Prove that a product of Hausdorff spaces is Hausdorff.
14. Define normal space and give an example. **(6×2=12)**

SECTION – C

Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks.

15. Prove that in any metric space X, each closed sphere is a closed set.
16. Let X be a metric space. Then prove that any finite intersection of open sets in X is open.
17. Let X and Y be metric spaces and f a mapping of X into Y. Prove that if f is continuous, then $f^{-1}(G)$ is open in X whenever G is open in Y.
18. Let X be a topological space. Suppose that $\mathcal{C}$ is a collection of open sets of X such that for each open set U of X and each x in U, there is an element C of $\mathcal{C}$ such that $x \in C \subset U$. Then prove that $\mathcal{C}$ is a basis for the topology of X.
19. Prove that the collection $\mathcal{S} = \{\pi_1^{-1}(U) | U \text{ open in } X\} \cup \{\pi_2^{-1}(V) | V \text{ open in } Y\}$ is a subbasis for the product topology on X × Y.
20. Prove that the map f : X → Y is continuous if X can be written as the union of open sets U_α such that $f|U_\alpha$ is continuous for each α.
21. Prove that the image of a connected space under a continuous map is connected.
22. Let Y be a subspace of X. Prove that Y is compact if and only if every covering of Y by sets open in X contains a finite subcollection covering Y.
23. Let f : X → Y be a continuous map of the compact metric space (X, d_X) to the metric space (Y, d_Y). Then prove that f is uniformly continuous.

24. Let X be a topological space. Let one-point sets in X be closed. Then prove that X is regular if and only if given a point x of X and a neighbourhood U of x, there is a neighbourhood V of x such that $\bar{V} \subset U$.
25. Prove that every metrizable space is normal.
26. Prove that a subspace of a regular space is regular. **(8×4=32)**

SECTION – D

Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks.

27. Let X be a complete metric space and let Y be a subspace of X. Then prove that Y is complete if and only if it is closed.
28. Let Y be a subspace of X. Then prove that a set A is closed in Y if and only if it equals the intersection of a closed set of X with Y.
29. Prove that every compact subspace of a Hausdorff space is closed.
30. Suppose that X has a countable basis. Then prove that
    a) Every open covering of X contains a countable subcollection covering X.
    b) There exists a countable subset of X that is dense in X. **(2×6=12)**

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