Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Mathematics (Honours) Degree (CBCSS – Supplementary) Examination, April 2025
(2019 – 2020 Admissions)
Core Course
BHM602 : TOPOLOGY

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any four** questions out of the five questions. **Each** question carries **1** mark. **(4×1=4)**

1. State True or False : "Union of two topologies on a set is again a topology."
2. Give an open set in $\mathbb{R}$, which is open with respect to scattering topology but not with respect to usual topology.
3. Define embedding of a space X into another space Y.
4. Define compact set. Give an example of an infinite compact set in $\mathbb{R}$.
5. State True or False : "Every T_1 space is T_2."

SECTION – B

Answer **any six** questions out of the nine questions. **Each** question carries **2** marks. **(6×2=12)**

6. Show that usual topology is weaker than semi-open interval topology.
7. If $X = \{a, b, c\}$, let $\tau_1 = \{\phi, X, \{a\}, \{a, b\}\}$ and $\tau_2 = \{\phi, X, \{a\}, \{b, c\}\}$. Find the smallest topology containing τ_1 and τ_2.

P.T.O.

8. Consider $Y = [-1, 1]$ as a subspace of $\mathbb{R}$. Let $B = \left\{x : \frac{1}{2} < |x| < 1\right\}$. Is B an open set in Y ? Is B an open set in $\mathbb{R}$? Justify your answers.
9. Define open map. Give an example.
10. Let X and Y be two sets. Give a topology on Y such that all functions from X to Y are continuous irrespective of the topology on X.
11. State Lebesgue covering lemma.
12. Can [0, 1] homeomorphic with (0, 1) ? Justify your answer.
13. State Urysohn's lemma.
14. Define T_4-space. Is $\mathbb{R}$ with usual topology a T_4-space ?

SECTION – C

Answer **any eight** questions out of the twelve questions. **Each** question carries **4** marks. **(8×4=32)**

15. Prove that second countability is a hereditary property.
16. Prove that Sierpinski space is not obtainable from a pseudo metric.
17. Prove that intersection of topologies is again a topology.
18. Prove that composition of two continuous functions is continuous.
19. Prove that $\overline{A} = A \cup A'$.
20. Find out the dense subsets of discrete, indiscrete and cofinite spaces.
21. Let X be a compact space and suppose $f : X \to Y$ is continuous and onto. Prove that Y is compact.
22. If X_1 and X_2 are connected topological spaces, then prove that $X_1 \times X_2$ is connected in the product topology.
23. Prove that every second countable space is separable.

24. Prove that property of T_2 is hereditary.
25. Suppose that a space X has the property that for every closed subset A of X, every continuous real valued function on A has a continuous extension to X. Then show that X is normal.
26. Prove that every completely regular space is normal.

SECTION – D

Answer **any two** questions out of the four questions. **Each** question carries **6** marks. **(2×6=12)**

27. a) Define hereditary property. Give an example of a property which is not hereditary.
    b) Prove that metrisability is a hereditary property.
28. Let X be a space and $A \subset X$.
    a) Define interior of A. Show that int(A) is an open set.
    b) Define closure of A. Show that $\overline{A}$ is closed.
29. Prove that a subset of $\mathbb{R}$ is connected iff it is an interval.
30. Prove that all metric spaces are T_4.
