K25U 0987

Reg. No. : ....

Name : ....

IV Semester B.Sc. Honours in Mathematics Degree (C.B.C.S.S.-OBE – Regular/Supplementary/Improvement) Examination, April 2025
(2021 – 2023 Admissions)
4B14 BMH : ADVANCED REAL ANALYSIS

Time : 3 Hours Max. Marks : 60

PART – A

Answer **any 4** questions out of 5 questions. **Each** question carries **1** mark. **(4×1=4)**

1. Determine the points of continuity of the function $f(x) = \frac{x^2+2x+1}{x^2+1}, x \in \mathbb{R}$.
2. Define Reimann integral of a function.
3. Let $I = [0,4]$. Find the norm of the partition $P = (0, 1, 2, 4)$.
4. State Cauchy criterion for uniform convergence.
5. Define gamma function.

PART – B

Answer **any 6** questions out of 9 questions, **Each** question carries **2** marks. **(6×2=12)**

6. State Bolzano's intermediate value theorem.
7. Give an example of a uniformly continuous function.
8. Prove that every constant function on [a, b] is Reimann integrable.
9. State second form fundamental theorem of calculus.
10. Evaluate $\lim e^{-nx}$ for $x \in \mathbb{R}, x \geq 0$.
11. Determine the radius of convergence of the series $\sum a_n x^n$, where $a_n = \frac{x^n}{n!}$.

P.T.O.

12. Let $h_n(x) = \frac{x^2+nx}{n}$ and $h(x) = x$ for $x \in \mathbb{R}$. Prove that (h_n) does not converge uniformly on $\mathbb{R}$ to h.
13. Does the integral $\int_1^\infty \frac{1}{\sqrt{x}}\,dx$ converges ? Justify.
14. Prove that $\beta(m,n) = \beta(n,m)$.

PART – C

Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks. **(8×4=32)**

15. State and prove preservation of intervals theorem.
16. Define Lipschitz function. Prove that every Lipschitz function is uniformly continuous.
17. Prove that $f(x) = x^2$ is not uniformly continuous on $A = [0, \infty)$.
18. If $f, g \in R[a, b]$, Prove that $f + g \in R[a, b]$.
19. Let $h(x) = x$ on [0,1]. Using squeeze theorem, prove that h is Reimann integrable on [0,1].
20. Let $f \in R[a, b]$. Define $F(z) = \int_a^z f$ for $z \in [a, b]$. Prove that F satisfies Lipschitz condition.
21. Prove that a sequence (f_n) of bounded functions on $A \subseteq \mathbb{R}$ converges uniformly on A to f if and only if $\|f_n - f\|_A \to 0$.
22. Show that if (f_n), (g_n) converge uniformly on the set A to f, g respectively, then $(f_n + g_n)$ converges uniformly on A to f + g.
23. State and prove Cauchy Hadamard theorem.

24. Prove that $\int_{-\infty}^{-2} \frac{1}{1-x}\,dx$ diverges.
25. Prove that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$.
26. Evaluate $\int_0^1 x^7(1-x)^8\,dx$.

PART – D

Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks. **(2×6=12)**

27. State and prove maximum minimum theorem.
28. State squeeze theorem. Prove that every monotone real valued function on [a, b] is Reimann integrable.
29. Let (h_n) be a sequence of bounded functions on $A \subseteq \mathbb{R}$. Prove that (f_n) converges uniformly on A to a bounded function f if and only if for each $\in > 0$ there is a number $H(\in)$ in $\mathbb{N}$ such that for all $m, n \geq H(\in)$, then $\|f_m - f_n\| \leq \in$.
30. Evaluate $\int_0^\infty e^{-x^2}\,dx$.