Reg. No.: .....

Name : .....

IV Semester B.Sc. Honours in Mathematics Degree (C.B.C.S.S.-OBE -Regular/Supplementary/Improvement) Examination, April 2025 (2021 - 2023 Admissions) 4B14 BMH: ADVANCED REAL ANALYSIS

Time: 3 Hours

Max. Marks: 60

PART - A

Answer any 4 questions out of 5 questions. Each question carries 1 mark.

- 1. Determine the points of continuity of the function  $f(x) = \frac{x^2 + 2x + 1}{x^2 + 1}$ ,  $x \in \mathbb{R}$ .
- 2. Define Reimann integral of a function.
- Let I = [0,4]. Find the norm of the partition P = (0, 1, 2, 4). 4. State Cauchy criterion for uniform convergence.
  - Define gamma function.

Answer any 6 questions out of 9 questions, Each question carries 2 marks.

PART - B

6. State Bolzano's intermediate value theorem.

 $(6 \times 2 = 12)$ 

- Give an example of a uniformly continuous function.
- State second form fundamental theorem of calculus.

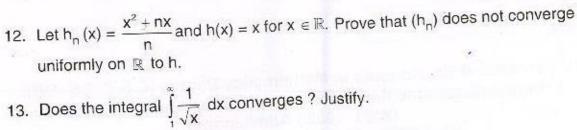
8. Prove that every constant function on [a, b] is Reimann integrable.

- 10. Evaluate  $\lim e^{-nx}$  for  $x \in \mathbb{R}$ ,  $x \ge 0$ .
- 11. Determine the radius of convergence of the series  $\sum a_n x^n$ , where  $a_n = \frac{x^n}{n!}$ .

P.T.O.

 $(8 \times 4 = 32)$ 

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14. Prove that  $\beta(m,n) = \beta(n,m)$ . PART-C

Answer any 8 questions out of 12 questions. Each question carries 4 marks.

State and prove preservation of intervals theorem. 16. Define Lipschitz function. Prove that every Lipschitz function is uniformly

continuous.

- 17. Prove that  $f(x) = x^2$  is not uniformly continuous on  $A = [0, \infty)$ . 18. If f,  $g \in R[a, b]$ , Prove that  $f + g \in R[a, b]$ .
- 19. Let h(x) = x on [0,1]. Using squeeze theorem, prove that h is Reimann
- integrable on [0,1]. 20. Let  $f \in R[a, b]$ . Define  $F(z) = \int f$  for  $z \in [a, b]$ . Prove that F satisfies Lipschitz
- condition. 21. Prove that a sequence  $(f_n)$  of bounded functions on  $A\subseteq \mathbb{R}$  converges uniformly on A to f if and only if  $\|f_n - f\|_A \to 0$ . 22. Show that if  $(f_n)$ ,  $(g_n)$  converge uniformly on the set A to f, g respectively, then
- $(f_n + g_n)$  converges uniformly on A to f + g. 23. State and prove Cauchy Hadamard theorem.

## 25. Prove that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ . 26. Evaluate $\int_{0}^{1} x^{7} (1-x)^{\theta} dx$ .

 $(2 \times 6 = 12)$ 

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Answer any 2 questions out of 4 questions. Each question carries 6 marks.

[a, b] is Reimann integrable.

State and prove maximum minimum theorem.

24. Prove that  $\int_{-\infty}^{-\infty} \frac{1}{1-x} dx$  diverges.

28. State squeeze theorem. Prove that every monotone real valued function on

- 29. Let  $(h_n)$  be a sequence of bounded functions on  $A \subseteq \mathbb{R}$ . Prove that  $(f_n)$ converges uniformly on A to a bounded function f if and only if for each ∈> 0
- there is a number  $H(\epsilon)$  in N such that for all m,  $n \ge H(\epsilon)$ , then  $||f_m f_n|| \le \epsilon$ . Evaluate ∫e<sup>-x²</sup>dx.

PART - D