K25U 0994

Reg. No. : ....

Name : ....

IV Semester B.Sc. Honours in Mathematics Degree (C.B.C.S.S. – Supplementary) Examination, April 2025
(2019 and 2020 Admissions)
BHM403 : COMPLEX ANALYSIS, FOURIER SERIES AND PARTIAL DIFFERENTIAL EQUATIONS

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any 4** questions out of 5 questions. **Each** question carries **1** mark. **(4×1=4)**

1. Write the standard form of Two dimensional wave equation.
2. Define connected set.
3. Find the accumulation point of the set $z_n = \frac{i}{n}$ (n = 1, 2, 3,....).
4. Define order of the PDE.
5. Find the fundamental period of sin 2πx.

SECTION – B

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks. **(6×2=12)**

6. Find all the values of z such that $e^z = -2$.
7. Is the statement "Arg (z_1z_2) = Arg z_1 + Arg z_2" always true ? Justify your answer.
8. Prove that if $\lim_{z \to z_0} f(z) = w_0$ then $\lim_{z \to z_0} |f(z)| = |w_0|$.
9. Define domain and give an example.
10. Solve for u given that $u_{yy} = 0$.
11. Write the Fourier series of an odd function of period 2L.

P.T.O.

12. If f(x) has a period p then prove that the period of $f\left(\frac{x}{b}\right)$, $b \neq 0$ is bp.
13. Is the function $f(x) = e^{-4x}$ $(-\pi < x < \pi)$ even, odd, neither even nor odd ? Justify your answer.
14. Define singular point of a function and find the singular points of
$$f(z) = \frac{z^3 + 4}{(z^2 - 3)(z^2 + 1)}.$$

SECTION – C

Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks. **(8×4=32)**

15. Prove that $|e^{-2z}| < 1$ if and only if Re z > 0.
16. If a function f(z) has a limit at point then prove that it is unique.
17. Find the points where the function $f(z) = |z|^2$ is differentiable.
18. Find the harmonic conjugate of the function u(x, y) = sinh x sin y.
19. Check whether the function f(z) = (3x + y) + i(3y – x) is entire or not.
20. Find the image of the vertical and horizontal segments, under the transformation $w = e^z$.
21. Find all the roots of the equation sin z = cosh4.
22. Prove the following :
    a) $\int_{-\pi}^{\pi} \cos nx \cos mx \, dx = 0$ $(n \neq m)$. Here n and m are integers.
    b) $\int_{-\pi}^{\pi} \sin nx \sin mx \, dx = 0$ $(n \neq m)$. Here n and m are integers.
    c) $\int_{-\pi}^{\pi} \sin nx \cos mx \, dx = 0$ $(n \neq m$ or $n = m)$. Here n and m are integers.

23. Find the Fourier series of the function $f(x) = \begin{cases} -k \text{ if } -2 < x < 0 \\ k \text{ if } 0 < x < 2 \end{cases}$, p = 2L = 4.
24. Solve the PDE $u_{xy} = u_x$.
25. Find a two dimensional Poisson equation whose solution is $u = \frac{1}{\sqrt{x^2 + y^2}}$.
26. Solve the system of PDEs $u_{xx} = 0$, $u_{yy} = 0$.

SECTION – D

Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks. **(2×6=12)**

27. Find the square roots of $\sqrt{3} + i$.
28. Find the type and transform into normal form and solve $u_{xx} - 2u_{xy} + u_{yy} = 0$.
29. Prove "A function f(z) = u(x, y) + iv(x, y) is analytic in a domain D if and only if v is a harmonic conjugate of u".
30. Find the two half range expansions of the function
$$f(x) = \begin{cases} \frac{2k}{L}x & \text{if } 0 < x < L/2 \\ \frac{2k}{L}(L - x) & \text{if } \frac{L}{2} < x < L. \end{cases}$$

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