IV Semester B.Sc. Honours in Mathematics Degree (C.B.C.S.S. – OBE-Regular/Supplementary/Improvement) Examination, April 2025
(2021 – 2023 Admissions)
4B16BMH : INTRODUCTION TO PARTIAL DIFFERENTIAL EQUATIONS

Time : 3 Hours   Max. Marks : 60

SECTION – A

Answer **any four** questions out of five questions. **Each** question carries
1 mark.   (4×1=4)

1. Write Laplace equation.

2. Find the order of the partial differential equation, $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0$.

3. Give an example of a non-linear partial differential equation of order 3.

4. Write a second order partial differential equation with variable coefficients.

5. Write the Lagrange's auxiliary equations for the partial differential equation
   $Pp + Qq = R$.

SECTION – B

Answer **any six** questions out of nine questions. **Each** question carries **2** marks.
                                                                                               (6×2=12)

6. Find the degree and order of the partial differential equation,
   $$\left\{\left(\frac{\partial^2 z}{\partial x^2}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2\right\} \cdot y = \left(\frac{\partial z}{\partial x}\right)^5 z^2.$$

7. State the principle of superposition of solutions of partial differential equation.

8. By eliminating arbitrary constants a and b, find the partial differential equation
   from $z = ax + by + a^2 + b^2$.

9. Define general integral of a partial differential equation. How can you find a
   singular integral of a partial differential equation ?

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10. Find the complete integral of $p^2 + q^2 = m^2$.

11. Solve $xys = 1$.

12. Classify the differential equation $x^2 u_{xx} + 2xy u_{xy} + y^2 u_{yy} = 0$.

13. Solve the differential equation $2r + 5s + 2t = 0$.

14. Solve the differential equation $(D^3 - 2D^2 D' + D(D')^2) z = 0$.

SECTION – C

Answer **any 8** questions out of 12 questions. **Each** question carries 4 marks.
                                                                                               (8×4=32)

15. Find the partial differential equation by eliminating arbitrary function from
    $f(x + yz, x^2 + y^2 - z^2) = 0$.

16. Solve $x^2(y - z)p + y^2(z - x)q = z^2(x - y)$.

17. Solve $p - 2q = 3x^2 \sin(y + 2x)$.

18. Find the complete integral of $(x + y)(p + q)^2 + (x - y)(p - q)^2 = 1$.

19. Find the singular integral of $p^3 + q^3 = 216z$.

20. Solve the differential equation $px + qy + pq = 0$.

21. Solve $s - t = \frac{x}{y^2}$.

22. Solve the partial differential equation $p + r + s = 1$.

23. Find the region in the xy-plane in which the following equation is
    hyperbolic $[(x - y)^2 - 1]u_{xx} + 2u_{xy} + [(x - y)^2 - 1]u_{yy} = 0$.

24. Solve $\frac{\partial^2 z}{\partial x^2} = a^2 z$, given that $\frac{\partial z}{\partial x} = a \sin y$ and $\frac{\partial z}{\partial y} = 0$ when $x = 0$.

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25. Find a real function V of x and y, reducing to zero when y = 0 and satisfying
    $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = -4\pi(x^2 + y^2)$.

26. Solve $(D^2 - DD' - 2(D')^2)z = (y - 1)e^x$.

SECTION – D

Answer **any 2** questions out of four questions. **Each** question carries **6** marks.
                                                                                               (2×6=12)

27. Find the integral of the partial differential equation
    $(x - y)y^2 p + (y - x)^2 xq = (x^2 + y^2)z$ which passes through the curve
    $xz = a^3, y = 0$.

28. Apply Jacobi's method to find the complete integral of $p_1 x_1 + p_2 x_2 = p_3^2$.

29. Solve the differential equation $r + (a + b)s + abt = xy$ using Monge's
    method.

30. Solve $(D^2 - (D')^2 - 3D + 3D')z = xy + e^{x+2y}$.

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