Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree (C.B.C.S.S. – OBE – Regular/Supplementary/ Improvement) Examination, April 2025
(2019 to 2022 Admissions)
CORE COURSE IN STATISTICS
6B11STA : Mathematical Methods for Statistics – III

Time : 3 Hours

Max. Marks : 48

PART – A

Answer **all** questions. **Each** question carries 1 mark. (6×1=6)

1. Define analytic function.
2. State a set of necessary conditions for a function to be analytic.
3. State Cauchy's theorem.
4. State Morera's theorem.
5. Define Taylor's series.
6. Define removable singularity.

PART – B

Answer **any seven** questions. **Each** question carries 2 marks. (7×2=14)

7. Examine whether the function $f(z) = \overline{Z}$ is analytic.
8. Show that the function u = cos (x) cosh (y) is harmonic.
9. Prove that an analytic function with constant real part is constant.
10. Find $\int_C \frac{z^2 + 5z + 6}{z - 2} dz$, where C is the circle of radius 1 and centre at the origin.
11. Find the singularity and its type of the function $f(z) = \frac{1}{\sin\left(\frac{1}{z}\right)}$.
12. Explain pole with the help of an example.

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13. What do you mean by radius of convergence of a power series.
14. State Jordan's lemma.
15. Find the residue of $f(z) = \frac{z^3}{(z-1)^4 (z-2)(z-3)}$ at z = 3.

PART – C

Answer **any four** questions. **Each** question carries 4 marks. (4×4=16)

16. Show that the function $f(z) = u + iv = \begin{cases} \frac{xy(x+iy)}{x^2 + y^2}, & x+iy \neq 0 \\ 0, & x+iy = 0 \end{cases}$ satisfies
    Cauchy-Riemann equations at the origin, but not analytic at the point.
17. State and prove Cauchy's integral formula.
18. $\int_C \frac{e^{5z}}{z - \pi i} dz$, where C is the ellipse $|z - 2| + |z + 2| = 6$.
19. Explain radius of convergence of a power series. Find the radius of convergence of the power series $\sum_{n=0}^{\infty} (\log n)^n z^n$.
20. State and prove Cauchy's residue theorem.
21. Evaluate $\int_C \frac{e^z}{z(z-1)^2} dz$, where C is $|z| = 2$.

PART – D

Answer **any two** questions. **Each** question carries 6 marks. (2×6=12)

22. Explain any one method of constructing analytic function with help of an example.
23. State and prove Poisson integral formula.
24. Find the Laurent series expression of $f(z) = \frac{1}{z^2 - 3z + 2}$ for $|z| > 2$.
25. Evaluate $\int_0^{2\pi} \frac{d\theta}{5 + 4\cos\theta}$.