Reg. No. : ....

Name : ....

I Semester M.Sc. Degree (CBCSS – OBE – Reg./Supple./Imp.) Examination, October 2024
(2023 Admission Onwards)
Mathematics/Mathematics (Multivariate Calculus and Mathematical Analysis, Modelling and Simulation, Financial Risk Management)
MSMAF01C01/MSMAT01C01 : ABSTRACT ALGEBRA

Time : 3 Hours Max. Marks : 80

PART – A

Answer **any five** questions. **Each** question carries **4** marks.

1. List all abelian groups, upto isomorphism, of order 360.
2. List the elements of $\mathbb{Z}_2 \times \mathbb{Z}_4$. Find the order of each of the elements. Is this group cyclic ?
3. Compute all Sylow 3-subgroups of S_4.
4. State butterfly lemma.
5. Find the center of $S_3 \times \mathbb{Z}_4$.
6. Express the polynomial $x^4 + 4$ as a product of linear factors in $\mathbb{Z}_5[x]$. **(5×4=20)**

PART – B

Answer **any three** questions. **Each** question carries **7** marks.

7. Show that every group of order p^2 is abelian.
8. State Sylow's third theorem. Prove that no group of order 30 is simple.
9. Let F be a field of quotients of D and let L be any field containing D. Prove that there exists a map $\psi : F \to L$ that gives an isomorphism of F with a subfield of L such that $\psi(a) = a$ for $a \in D$.

P.T.O.

10. State and prove second isomorphism theorem.
11. An ideal $\langle p(x) \rangle \neq \{0\}$ of F[x] is maximal if and only if p(x) is irreducible over F. **(3×7=21)**

PART – C

Answer **any three** questions. **Each** question carries **13** marks.

12. a) Let $G_1, G_2, \ldots, G_n$ be groups. Prove that $\prod_{i=1}^{n} G_i$ is a group under the operation $(a_1, a_2, \ldots, a_n)(b_1, b_2, \ldots, b_n) = (a_1b_1, a_2b_2, \ldots, a_nb_n)$.
    b) Find the order of (3, 10, 9) in $\mathbb{Z}_4 \times \mathbb{Z}_{12} \times \mathbb{Z}_{15}$.
    c) Let X be a G-set. For $x_1, x_2 \in X$, let $x_1 \sim x_2$ if and only if there exists $g \in G$ such that $gx_1 = x_2$. Prove that $\sim$ is an equivalence relation.
13. State and prove first Sylow theorem.
14. a) If H and K are finite subgroups of a group G, prove that $|HK| = \frac{(|H|)(|K|)}{|H \cap K|}$.
    b) Prove that any two fields of quotients of an integral domain D are isomorphic.
15. a) Prove that two subnormal series of a group G have isomorphic refinements.
    b) Prove that S_3 is solvable.
16. a) State and prove division algorithm for F[x].
    b) Let $f(x) = x^4 - 3x^3 + 2x^2 + 4x - 1$ and $g(x) = x^2 - 2x + 3$ be two polynomials in $\mathbb{Z}_5[x]$. Find q(x) and r(x) such that f(x) can be written as $f(x) = q(x)g(x) + r(x)$. **(3×13=39)**

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