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# K24P 3937

Reg. No. : ................................

Name : ....................................

**I Semester M.Sc. Degree (C.B.C.S.S. – O.B.E. – Reg./Supple./Imp.) Examination, October 2024**
**(2023 Admission Onwards)**
**Mathematics/Mathematics (Multivariate Calculus and Mathematical Analysis, Modelling and Simulation, Financial Risk Management)**
**MSMAF01C02/MSMAT01C02 : LINEAR ALGEBRA**

Time : 3 Hours Max. Marks : 80

PART – A

Answer **any five** questions. **Each** question carries **4** marks.

1. Define linear transformation. Let $T : R^2 \to R^2$ be defined by $T(x, y) = (1 + x, y)$. Is T linear ? Justify.
2. Find the range and rank of the identity transformation on $R^2$.
3. Define dual space. If V is a vector space of dimension n, find the dimension of the dual space $V^*$.
4. Let T be a linear operator on $R^2$ defined by $T(x, y) = (-y, x)$. Find the characteristic values of T.
5. State Cyclic decomposition theorem.
6. Let V be an inner product space. Prove that $|(\alpha/\beta)| \leq \|\alpha\|\|\beta\|$ for any vectors $\alpha, \beta \in V$. **(5×4=20)**

PART – B

Answer **any three** questions. **Each** question carries **7** marks.

7. Let V, W be vector spaces over the field F and L(V, W) be the set of all linear transformations from V into W. Prove that L(V, W) is a vector space over F with respect to the addition and scalar multiplication defined by $(T + U)(\alpha) = T\alpha + U\alpha$ and $(cT)(\alpha) = c(T\alpha)$, where $T, U \in L(V,W)$ and $c \in F$.

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8. Let T be the linear operator on $R^3$ defined by $T(x_1, x_2, x_3) = (3x_1, x_1 - x_2, 2x_1 + x_2 + x_3)$. Is T invertible ? If so find $T^{-1}$.
9. Let V be a finite dimensional vector space over the field F. For each $\alpha$ in V define $L_\alpha(f) = f(\alpha)$, f in $V^*$. Prove that the mapping $\alpha \to L_\alpha$ is an isomorphism of V onto $V^{**}$.
10. Let V be a finite dimensional vector space over the field F and let T be a linear operator on V. Prove that T is diagonalizable if and only if the minimal polynomial of T has the form $p = (x - c_1) \ldots (x - c_k)$, where $c_1, \ldots, c_k$ are distinct elements of F.
11. Let T be a linear operator on a finite dimensional vector space V over the field F. Suppose that a minimal polynomial for T decomposes over F into a product of linear functionals. Then prove that there is a diagonalizable operator D on V and a nilpotent operator N on V such that
    i) $T = D + N$, ii) $DN = ND$.

    Also show that the diagonalizable operator D and the nilpotent operator N are uniquely determined by (i) and (ii) and each of them is a polynomial in T. **(3×7=21)**

PART – C

Answer **any three** questions. **Each** question carries **13** marks.

12. a) Let V be an n-dimensional vector space over the field F, and let W be an m-dimensional vector space over F. Prove that the space L(V, W) is finite dimensional and has dimension mn.
    b) Prove that every n-dimensional vector space over the field F is isomorphic to the space $F^n$.
13. Let $\mathcal{B} = \{\alpha_1, \alpha_2, \alpha_3\}$ be basis for $C^3$ defined by $\alpha_1 = (1, 0, -1)$, $\alpha_2 = (1, 1, 1)$, $\alpha_3 = (2, 2, 0)$. Find the dual basis of $\mathcal{B}$.
14. Let T be a linear operator on $R^3$ which is represented in the standard ordered basis by the matrix $\begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$. Prove that T is diagonalizable by exhibiting a basis for $R^3$, each vector of which is a characteristic vector of T.
15. Let T be a linear operator on a finite dimensional vector space V. If f is the characteristic polynomial for T, prove that $f(T) = 0$.
16. State and prove Gram Schmidt orthogonalization process. **(3×13=39)**

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