Unit – III

13. a) Let C be a connected oriented plane curve and let $\beta : I \to C$ be a unit speed global parametrization of C. Then prove that β is either one to one or periodic. Also prove that β is periodic if and only if C is compact.

    b) Let $\alpha : I \to \mathbb{R}^{n+1}$ be a parametrized curve and if $\beta : I \to \mathbb{R}^{n+1}$ is a reparametrization of α then prove that $l(\alpha) = l(\beta)$.

14. a) Find the principal curvatures at p and the principal curvature directions at p of the hyperboloid $-x_1^2 + x_2^2 + x_3^2 = 1$ in $\mathbb{R}^3$ at $p = (0, 0, 1)$.

    b) Find the Gaussian curvature of the ellipsoid $\frac{x_1^2}{a^2} + \frac{x_2^2}{b^2} + \frac{x_3^2}{c^2} = 1$, a, b, c all $\neq 0$ oriented by its outward normal.

15. a) State and prove the inverse function theorem for n-surfaces.

    b) Let S be an n-surface in $\mathbb{R}^{n+1}$ and let $f : S \to \mathbb{R}^k$. Then prove that f is smooth if and only if $f \circ \varphi : U \to \mathbb{R}^k$ is smooth for each local parametrization $\varphi : U \to S$.
