Reg. No.:

Name :

Sixth Semester B.Sc. Degree (C.B.C.S.S. - O.B.E. - Regular/ Supplementary/Improvement) Examination, April 2025 (2019 to 2022 Admissions) CORE COURSE IN MATHEMATICS

6B10 MAT : Real Analysis - II

Time: 3 Hours

Max. Marks: 48

PART - A

Answer any four questions, Each question carries one mark. Give an example of a Lipschitz Function.

 $(4 \times 1 = 4)$

- 2. Define Riemann integral of a function $f : [a, b] \rightarrow \mathbb{R}$.
- 3. State Additivity Theorem. Define Beta function.
- 5. Find $\lim_{n\to\infty} \frac{x}{x+n}$ for all $x \in [0, 1]$.

Answer any eight questions. Each question carries two marks.

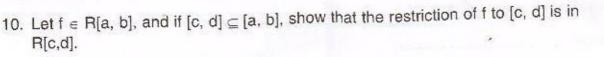
PART - B

(8×2=16)

- 6. Define step function on [a, b]. Give an example of a step function on [0, 2]. 7. Check whether $f(x) = \sin x$ is Lipschitz on \mathbb{R} . Justify your answer.
- 8. Let $f(x) = x^3$ for $x \in [0, 4]$, calculate the Riemann sum with respect to the partition P = (0, 1, 2, 4), take tags at the left end point of the subintervals.
- 9. Show that every constant function on [a, b] is in $\mathcal{R}[a, b]$.

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- 11. Show that $\int_0^1 \frac{1}{1-x} dx$ diverges. 12. Show that $\Gamma n = (n-1) \Gamma(n-1)$.
- Compute Γ(-3/2).
- 14. Let $f_n:\mathbb{R}\to\mathbb{R}$ be defined by $f_n(x)=\frac{sinnx}{n}$. Find the pointwise limit of the sequence of functions (f_n) .
- 15. Define a metric d on a set S. 16. State Dini's theorem.
- Answer any four questions. Each question carries four marks.

continuous on (0, 1).

17. Define uniformly continuous function. Show that $\sin\left(\frac{1}{x}\right)$ is not uniformly continuous on (0, 1)

18. Show that every continuous function on [a, b] is Riemann integrable. 19. Let f, g \in R[a, b], if f(x) \leq g(x) for all x \in [a, b] then show that $\int_a^b f \leq \int_a^b g$.

- 20. Evaluate $\int_0^3 \frac{dx}{(x-1)^{2/3}}$
- 23. Let (f_n) be a sequence of continuous functions on a set $A \subseteq \mathbb{R}$ and suppose that (f_n) converges uniformly on A to a function $f:A\to\mathbb{R}.$ Show that f is

continuous on A.

25. State and prove fundamental theorem of calculus (Second form). 26. a) Show that $\int_{1}^{\infty} \frac{1}{x^{p}} dx = \begin{cases} \frac{1}{p-1}, & p > 1 \\ \infty, & p \le 1 \end{cases}$

b) Investigate the convergence of $\int_1^{\infty} \frac{1-e^{-x}}{x} dx$. 27. Let (f_n) be a sequence of functions in $\mathcal{R}[a,b]$ and suppose that (f_n) converges uniformly on [a, b] to f. Show that $f \in \mathcal{R}[a, b]$ and $\int_a^b f = \lim_{n \to \infty} \int_a^b f_n$.

Answer any two questions. Each question carries six marks.

b) Show that $f(x) = \frac{e^x - 1}{x}$ is uniformly continuous on (0,1).

function is continuous on [a, b].

21. Show that $\Gamma n\Gamma(1-n) = \frac{\pi}{\sin n\pi}$. 22. Evaluate $\int_{0}^{\pi/2} (\sin x)^{8/3} (\sec x)^{1/2} dx$.

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PART - D

only if it can be defined at the endpoints a and b such that the extended

24. a) Show that a function f is uniformly continuous on the interval (a, b) if and

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 $(2 \times 6 = 12)$