K25U 0830

Reg. No. : .....

Name : .....

IV Semester B.Sc. Degree (C.B.C.S.S.-OBE – Regular/Supplementary/ Improvement) Examination, April 2025
(2019 to 2023 Admissions)
CORE COURSE IN MATHEMATICS
4B04MAT : Number Theory and Applications of Integrals

Time : 3 Hours Max. Marks : 48

PART – A

Answer **any four** questions. **Each** question carries **one** mark. **(4×1=4)**

1. State Fundamental theorem on arithmetic.
2. Define relatively prime integers.
3. Find gcd(–27, –35).
4. Evaluate $\int_0^3 \sqrt{1+y}\, dy$.
5. Define arc length.

PART – B

Answer **any eight** questions. **Each** question carries **2** marks. **(8×2=16)**

6. Let a, b, c and d be integers. If a/b and c/d, then prove that ac/bd.
7. For any integer $k \neq 0$, prove that $\gcd(ka, kb) = |k| \gcd(a, b)$.
8. If p is a prime and p/ab, then prove that p/a or p/b.
9. For arbitrary integers a and b, prove that $a \equiv b \pmod n$ if and only if a and b leave same nonnegative remainder when divided by n.
10. Let $n > 0$ be fixed and a, b, c and d be arbitrary integers. If $a \equiv b \pmod n$ and $c \equiv d \pmod n$, then prove that $a + c \equiv b + d \pmod n$.
11. If a is odd integer, prove that $a^2 \equiv 1 \pmod 8$.

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12. Evaluate $\int_0^1 t^3(1+t^4)^3\, dt$.
13. Evaluate $\int_{\pi/6}^{\pi/3} (1-\cos 3t)\sin 3t\, dt$.
14. Find the area of the region enclosed by the curves $4x^2 + y = 4$ and $x^4 - y = 1$.
15. The circle $x^2 + y^2 = a^2$ is rotated about the x-axis to generate a sphere. Find its volume.
16. Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$ and the lines $y = 2$ and $x = 0$ about x-axis.

PART – C

Answer **any four** questions. **Each** question carries **four** marks. **(4×4=16)**

17. Let a, b be integers not both zero. For a positive integer d, $d = \gcd(a, b)$ if and only if
    a) d/a and d/b.
    b) Whenever c/a and c/b, then c/d.
18. Determine solutions in the integers of the Diophantine equation $56x + 72y = 40$.
19. State Wilson's theorem. Is the converse of this theorem true ? Justify.
20. Find the remainder when 15! is divided by 17.
21. Find the length of the graph of $f(x) = \frac{x^3}{12} + \frac{1}{x}, 1 \le x \le 4$.
22. Find the area inside the cardioid $r = a(1 + \cos\theta)$, $a > 0$.
23. Find the area of the surface generated by revolving the curve $y = 2\sqrt{x}, 1 \le x \le 2$ about the x-axis.

PART – D

Answer **any two** questions. **Each** question carries **six** marks. **(2×6=12)**

24. Use Euclidean algorithm to obtain integers x and y satisfying $\gcd(12378, 3054) = 12378x + 3054y$.
25. Use Fermat's method to factor 119143.
26. Find the length of the cardioid $r = 1 + \cos\theta$.
27. Find the area of the surface generated by revolving about the x-axis the portion of the astroid $x^{2/3} + y^{2/3} = 1$ in the first quadrant.

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