K25U 0187

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree (C.B.C.S.S. – OBE – Regular/Supplementary/ Improvement) Examination, April 2025
(2019 to 2022 Admissions)
DISCIPLINE SPECIFIC ELECTIVE IN STATISTICS
6B13CSTA : Stochastic Processes

Time : 3 Hours Max. Marks : 48

PART – A
(Short Answer)

Answer **all** questions. **1** mark **each.**

1. State consistency theorem.
2. Define probability generating function of a discrete random variable.
3. Give an example for a transition probability matrix.
4. Define a Markov process.
5. Define Gaussian process.
6. What is meant by probability of ultimate extinction ? **(6×1=6)**

PART – B
(Short Essay)

Answer **any 7** questions. **2** marks **each.**

7. The pgf of a random variable is $P(s)=\frac{(1+s)^2}{4}$. Write down the pmf of the random variable and its mean.
8. Distinguish between parameter space and state space with suitable examples.
9. Define conditional expectation and conditional variance of a pair of random variables.
10. State Chapman Kolmogorov equation. Mention its use.
11. Define first passage time distribution of a Markov chain.
12. Define Poisson process.

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13. Give any two properties of Poisson process.
14. Define Compound Poisson Process.
15. Distinguish between wide sense stationary process and strict sense stationary process. **(7×2=14)**

PART – C
(Essay)

Answer **any 4** questions. **4** marks **each.**

16. Define a stochastic process. How will you classify it ?
17. The single step transition probability matrix of a Markov chain is $P=\begin{bmatrix}0 & 1 & 0\\ \frac{1}{2} & 0 & \frac{1}{2}\\ 0 & 1 & 0\end{bmatrix}$. Find the three step transition probabilities.
18. Prove that the relation communication in a Markov chain is an equivalence relation.
19. Prove that sum of two independent Poisson processes is again a Poisson process.
20. Suppose in a branching process the pgf of the offspring distribution is $\frac{1+s+2s^2}{4}$. Find the probability of ultimate extinction.
21. Consider the stochastic process defined by $X(t) = U\cos(\omega t) + V\sin(\omega t)$, where U and V are uncorrelated random variables each with mean 0 and variance 1 and ω is a positive constant. Show that $\{X(t)\}$ is covariance stationary. **(4×4=16)**

PART – D
(Essay)

Answer **any 2** questions. **6** marks **each.**

22. The joint pdf of (X, Y) is $f(x, y) = kxy$, $1 \le x \le y \le 2$. Find (i) value of k (ii) marginal pdf of X and Y and (iii) conditional pdf of X given Y and Y given X.
23. What are the different classifications of states of a Markov chain ? Classify the states of a Markov chain with transition probability matrix $\begin{bmatrix}0 & 1 & 0\\ 0.75 & 0 & 0.25\\ 0 & 1 & 0\end{bmatrix}$.
24. Let $\{X(t), t\in T\}$ be a Poisson process. Find (i) the distribution of inter arrival time (ii) conditional distribution of X(s) given X(t) for s<t.
25. Derive the relation between the probability generating functions of the offspring distribution and Galton-Watson branching process. **(2×6=12)**

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