K25U 0161

Reg. No.: .....

Name : .....

Sixth Semester B.Sc. Degree (C.B.C.S.S. - OBE - Regular/Supplementary/ Improvement) Examination, April 2025 (2019 to 2022 Admissions) CORE COURSE IN MATHEMATICS

6B13 MAT : Linear Algebra

Time: 3 Hours

Max. Marks: 48

### PART - A

Answer any four questions. Each question carries one mark. Define vector space.

 $(4 \times 1 = 4)$ 

- 2. What is the dimension of the vector space of all 2 x 2 symmetric matrices
- over R? 3. Suppose that  $T: \mathbb{R}^2 \to \mathbb{R}^2$  is linear, T(1, 0) = (1, 4), and T(0, 1) = (2, 5).
- What is T(2, 3) ?
- 4. Find the eigenvalues of the matrix A = 0 2 6 0 0 5 5. Let A be a 3 × 3 matrix, with eigenvalues 1, 0, 2. Find determinant of A.
- PART B

### Answer any eight questions. Each question carries two marks.

6. Let V be a vector space over F. Show that for each element x in V, there exist

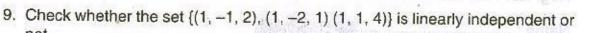
 $(8 \times 2 = 16)$ 

- a unique element y in V such that x + y = 0. 7. Let  $M_{n \times n}$  ( $\mathbb{F}$ ) be the set of all  $n \times n$ -matrices over  $\mathbb{F}$ . Show that
- $S = \{A \in M_{n \times n}(\mathbb{F}) \mid tr(A) = 0\}$  is a subspace of  $M_{n \times n}(\mathbb{F})$ . 8. Let V be a vector space over  $\mathbb{F}$ . Show that 0.x = 0 for each  $x \in V$ .

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- 10. Let  $V = M(2, \mathbb{R})$ , the set of all  $2 \times 2$ -matrices over  $\mathbb{R}$  and let
- $W = \left\{ A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \in V | a_{11} + a_{12} = 0 \right\}. \text{ Find a basis of W.}$ 11. Find the rank of a matrix A, where A = 3
- 12. Show that  $T: \mathbb{R}^3 \to \mathbb{R}^2$  defined by T(x, y, z) = (x, x + y) is a linear transformation. 13. Explain the condition for consistency and nature of solution of a
- non-homogeneous linear system of equations Ax = B. 14. Let A be a  $2 \times 3$  matrix and B be a  $3 \times 3$  matrix with rank (A) = 2 and rank(B) = 3. Find rank(AB).
- 15. Find the characteristic equation of the matrix A =16. Prove that the eigenvalues of an idempotent matrix are either zero or unity.
- PART C Answer any four questions. Each question carries four marks.

## 17. Using example, show that union of two subspaces of a vector space need not

18. If V is a vector space generated by a finite set S, then show that some subset of S is a basis for V.

- 19. Find a basis and dimension of the subspace  $W = \{A \in M_{2\times 2}(\mathbb{R}) | tr(A) = 0\}$ of  $M_{2\times 2}(\mathbb{R})$ . 20. Let  $T:V\to W$  be a linear transformation. Show that T is one-one if and only if

 $N(T) = \{0\}.$ 

be a subspace.

# basis is $A = \begin{bmatrix} 0 & 1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$ find T(x, y, z).

22. Let T be the linear operator on  $\mathbb{R}^3$ , the matrix of which in the standard ordered

 $(6 \times 2 = 12)$ 

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23. Verify Cayley-Hamilton theorem for the matrix  $A = \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$  and find its

21. Let  $T: P_2(\mathbb{R}) \to P_3(\mathbb{R})$  be a linear transformation defined by T(f(x)) = (2 - x)f(x).

Find the matrix of T with respect to the standard basis of P2 and P3.

24. Let W<sub>1</sub> and W<sub>2</sub> be subspaces of a vector space V. Prove that V is the direct sum of W1 and W2 if and only if each vector in V can be uniquely expressed as  $x_1 + x_2$ , where  $\tilde{x}_1 \in W_1$  and  $x_2 \in W_2$ . 25. Let S be a linearly independent subset of a vector space V, and let v be a vector

Answer any two questions. Each question carries six marks.

- in V that is not in S. Then show that  $S \cup \{v\}$  is linearly dependent if and only if v ∈ Space(S). 26. Let V and W be vector spaces and T : V  $\rightarrow$  W be linear. Prove that N(T) and R(T) are subspaces of V and W respectively.
- 27. Find the characteristic equation of the matrix  $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$  and hence find