Reg. No. : ....

Name : ....

VI Semester B.Sc. Mathematics (Honours) Degree (C.B.C.S.S. – OBE-Regular/Supplementary/Improvement) Examination, April 2025
(2021 and 2022 Admissions)
DISCIPLINE SPECIFIC ELECTIVE COURSE
6B27C BMH : Fuzzy Mathematics

Time : 3 Hours | Max. Marks : 60

I. Answer **any 4** out of 5 questions. **Each** question carries **1** mark. **(4×1=4)**

1) Define a membership function.
2) Define a normal fuzzy set.
3) Find the α – cuts with $\alpha = 0.2$ of the fuzzy set $A = 0.2/x_1 + 0.4/x_2 + 0.6/x_3 + 0.8/x_4 + 1/x_5$.
4) Define extension principle.
5) Find [2, 5] + [1, 3].

II. Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks. **(6×2=12)**

6) Define interval valued fuzzy set with an example.
7) Compute the scalar cardinalities of $A = 1/x + 1/y + 1/z$.
8) Determine the mathematical formulas and graphs of the membership grade function of $\bar{A}$ and $A \cup B$.
9) State the first decomposition theorem.
10) State Characterization theorem of t – norms.
11) Write the Pseudo inverse of the increasing generator $g_1(a) = a^p (p > 0)$ for any $a \in [0,1]$.
12) Prove that the triples $\langle \min, \max, c \rangle$ and $\langle i_{min}, U_{max}, c \rangle$ are dual with respect to the fuzzy complement c.
13) Prove that $A.(B + C) \subseteq A.B + A.C$.
14) Find [3, 4]. [2, 2].

P.T.O.

III. Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks. **(8×4=32)**

15) Prove that a fuzzy set A on R is convex if and only if $A(\lambda x_1 + (1-\lambda) x_2) \geq \min[A(x_1), A(x_2)]$ for all $x_1, x_2 \in R$ and all $\lambda \in [0,1]$, where minimum denotes the minimum operator.
16) For any pair of fuzzy subset defined on a finite universal set X, the degree of subsethood,

$$S(A, B) = \frac{1}{|A|}\left(|A| - \sum_{x \in X} \max[0, A(x) - B(x)]\right).$$ Then prove that

$$S(A, B) = \frac{|A \cap B|}{|A|}.$$

17) For any $A \in F(X)$, prove that ${}^{\alpha}A = \cap_{\beta<\alpha} {}^{\beta}A = \cap_{\beta<\alpha} {}^{\beta+}A$.
18) State and prove the second decomposition theorem.
19) If c is a continuous fuzzy complement, then prove that c has a unique equilibrium.
20) Write the axiomatic skeleton for fuzzy intersection/t-norms.
21) Let i_{ω} denote the class of Yager t – norms defined by

$$i_{\omega}(a,b) = 1 - \min\left(1, [(1-a)^{\omega} + (1-b)^{\omega}]^{\frac{1}{\omega}}\right)(\omega > 0).$$ Then prove that $i_{min}(a,b) \leq i_{\omega}(a,b) \leq \min(a,b)$ for all $a, b \in [0,1]$.

22) Prove that the standard fuzzy union is the only idempotent t – conorm.
23) For $\alpha \to 0$, prove that the function $h_{\alpha}(a_1, a_2, \ldots, a_n) = \left(\frac{a_1^2 + a_2^2 + \ldots + a_n^2}{n}\right)^{\frac{1}{\alpha}}$ converges to the geometric mean $(a_1 a_2 \ldots a_n)^{\frac{1}{n}}$.
24) Explain linguistic variables with an example.
25) Let $A \in F(R)$ and A is a fuzzy number. Then prove that there exist a closed interval $[a,b] \neq \phi$ such that

$$A(x) = \begin{cases} 1 & \text{for } x \in [a,b] \\ l(x) & \text{for } x \in (-\infty, a) \\ r(x) & \text{for } x \in (b, \infty), \end{cases}$$

where l is a function from $(-\infty, a)$ to [0,1] that is monotonic increasing and continuous from the right and such that $l(x) = 0$ for $x \in (-\infty, \omega_1)$.

26) Let A and B be two fuzzy numbers whose membership functions are given by

$$A(x) = \begin{cases} \frac{x+2}{2} & \text{for } -2 < x \leq 0 \\ \frac{2-x}{2} & \text{for } 0 < x < 2 \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad B(x) = \begin{cases} \frac{x-2}{2} & \text{for } 2 < x \leq 4 \\ \frac{6-x}{2} & \text{for } 0 < x \leq 6 \\ 0 & \text{otherwise} \end{cases}.$$

Then calculate the fuzzy numbers A + B and A – B.

IV. Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks. **(2×6=12)**

27) Explain the characteristics and significance of the paradigm shift.
28) Let $f : X \to Y$ be an arbitrary function. Then for any $A \in F(X)$ and all $\alpha \in [0,1]$, prove that
    i) ${}^{\alpha+}[f(A)] = f({}^{\alpha+}A)$.
    ii) ${}^{\alpha}[f(A)] \supseteq f({}^{\alpha}A)$.
29) For all $a, b \in [0,1]$ prove that $\max(a,b) \leq U(a,b) \leq U_{max}(a,b)$.
30) Consider two triangle shape fuzzy numbers A and B defined as follows :

$$A(x) = \begin{cases} 0 & ; x < -1 \text{ and } x > 3 \\ \frac{x+1}{2} & ; -1 < x \leq 1 \\ \frac{3-x}{2} & ; 1 < x \leq 3 \end{cases} \quad \text{and} \quad B(x) = \begin{cases} 0 & ; x \leq 1 \text{ and } x > 5 \\ \frac{x-1}{2} & ; 1 < x \leq 3 \\ \frac{5-x}{2} & ; 3 < x \leq 5 \end{cases}$$

Then find ${}^{\alpha}A$ and ${}^{\alpha}B$.

Hence find,

a) ${}^{\alpha}(A+B)$
b) ${}^{\alpha}(A-B)$
c) ${}^{\alpha}(A.B)$
d) ${}^{\alpha}(A/B)$

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