| Reg. No. : | |
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| Name : | |

Sixth Semester B.Sc. Mathematics (Honours) Degree (C.B.C.S.S. – O.B.E. - Regular/Supplementary/Improvement) Examination, April 2025 (2021 and 2022 Admissions) Core Course

6B26 BMH: MEASURE THEORY

Time: 3 Hours

Max. Marks: 60

SECTION - A

Answer any 4 questions out of 5 questions. Each question carries 1 mark. (4x1=4)

- 1. If f is any function on X to R, define positive part of f. Define step function.
- 3. When do we say that a real valued function is simple? 4. When do we say that the indefinite integral of a function in L is countably
- additive? State Lebesgue Dominated Convergence Theorem.
- SECTION B

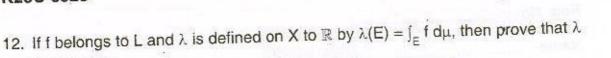
Answer any 6 questions out of 9 questions. Each question carries 2 marks. (6×2=12) Is the constant function on R measurable ? Justify your answer.

- 7. Define measurable space.
- 8. Prove that the sum of two complex valued measurable function is measurable.
- Define measure space.
- 10. Prove that if $m^*(A) = 0$, then $m^*(A \cup B) = m^*(B)$.
- 11. If f belongs to M⁺ and $c \ge 0$, then prove that cf belongs to M⁺ and $\int cf d\mu = c \int f d\mu$.

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- is a charge. 13. If f belongs to $M^+(X, X)$, then define the integral of f. Further if f belongs to M+(X, X) and E belongs to X, then define the integral of f over E.
- 14. If ϕ is a simple function in M⁺(X, X) and λ is defined for E in X by $\lambda(E) = \int \phi \chi_E d\mu$, then prove that λ is a measure on X.
- SECTION C Answer any 8 questions out of 12 questions. Each question carries 4 marks. (8×4=32)

15. Prove that if f is measurable then so is | f |. 16. Let B be the borel algebra on the set of real numbers. Prove that any monotone

function is Borel measurable.

 $\int f d\mu = \lim \int f_n d\mu$.

- 17. If X_1 and X_2 are σ -algebras of subsets of X. Then prove that $X_1 \cap X_2$ is a σ-algebra. 18. Prove that a countable set has outer measure zero.
- 19. Let μ be a measure defined on a σ -algebra on X. If (F_n) is a decreasing sequence in X and if $\mu(F_1) < +\infty$, then prove that $\mu\Big(\bigcap_{i=1}^\infty F_n\Big) = \lim_{i \to \infty} \mu(F_n)$.
- 20. Let A be the set of irrationals in the interval [0, 1]. Prove that $m^*(A) = 0$. 21. If f belongs to M+ and if λ is defined on X by $\lambda(E) = \int_E f d\mu$, then prove that λ is a measure. 22. If (f_n) is a monotone increasing sequence of functions in $M^+(X, X)$ which
- 23. State and prove Fatou's Lemma.

converges μ-almost everywhere on X to a function f in M+, then prove that

26. Prove that a measurable function f belongs to L if and only if | f | belongs to L.

on X if and only if $\int f d\mu = 0$.

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Also prove that $|\int f d\mu| \leq \int |f| d\mu$. SECTION - D

Answer any 2 questions out of 4 questions. Each question carries 6 marks. (2x6=12)

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24. If (fn) is a monotone increasing sequence of functions in M+(X, X) which

25. Suppose that f belongs to M⁺. Then prove that $f(x) = 0 \mu$ -almost everywhere

converges to f, then prove that $\int f \, d\mu = \lim \int f_n \, d\mu.$

- 27. Prove that f is measurable if and only if the positive and negative part of f is measurable. 28. Prove that the union of a finite collection of measurable sets is measurable.
- 29. If ϕ and Ψ are simple functions in M⁺(X, X), then prove that
- 30. Prove that a constant multiple of αf and a sum f + g of functions in L belongs
 - b) $\int (f + g) d\mu = \int f d\mu + \int g d\mu$.

a) $\int \alpha f d\mu = \alpha \int f d\mu$

to L and