K25U 0323

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Mathematics (Honours) Degree (C.B.C.S.S. – O.B.E. – Regular/Supplementary/Improvement) Examination, April 2025
(2021 and 2022 Admissions)
Core Course
6B26 BMH : MEASURE THEORY

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any 4** questions out of 5 questions. **Each** question carries **1** mark. **(4×1=4)**

1. If f is any function on X to **R**, define positive part of f.
2. Define step function.
3. When do we say that a real valued function is simple ?
4. When do we say that the indefinite integral of a function in L is countably additive ?
5. State Lebesgue Dominated Convergence Theorem.

SECTION – B

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks. **(6×2=12)**

6. Is the constant function on **R** measurable ? Justify your answer.
7. Define measurable space.
8. Prove that the sum of two complex valued measurable function is measurable.
9. Define measure space.
10. Prove that if $m^*(A) = 0$, then $m^*(A \cup B) = m^*(B)$.
11. If f belongs to M^+ and $c \geq 0$, then prove that cf belongs to M^+ and $\int cf\, d\mu = c \int f\, d\mu$.

P.T.O.

12. If f belongs to L and λ is defined on X to $\mathbb{R}$ by $\lambda(E) = \int_E f\, d\mu$, then prove that λ is a charge.
13. If f belongs to $M^+(X, X)$, then define the integral of f. Further if f belongs to $M^+(X, X)$ and E belongs to X, then define the integral of f over E.
14. If φ is a simple function in $M^+(X, X)$ and λ is defined for E in X by $\lambda(E) = \int \varphi\chi_E d\mu$, then prove that λ is a measure on X.

SECTION – C

Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks. **(8×4=32)**

15. Prove that if f is measurable then so is $|f|$.
16. Let B be the borel algebra on the set of real numbers. Prove that any monotone function is Borel measurable.
17. If X_1 and X_2 are σ-algebras of subsets of X. Then prove that $X_1 \cap X_2$ is a σ-algebra.
18. Prove that a countable set has outer measure zero.
19. Let μ be a measure defined on a σ-algebra on X. If (F_n) is a decreasing sequence in X and if $\mu(F_1) < +\infty$, then prove that $\mu\left(\bigcap_{n=1}^{\infty} F_n\right) = \lim \mu(F_n)$.
20. Let A be the set of irrationals in the interval [0, 1]. Prove that $m^*(A) = 0$.
21. If f belongs to M^+ and if λ is defined on X by $\lambda(E) = \int_E f\, d\mu$, then prove that λ is a measure.
22. If (f_n) is a monotone increasing sequence of functions in $M^+(X, X)$ which converges μ–almost everywhere on X to a function f in M^+, then prove that $\int f\, d\mu = \lim \int f_n d\mu$.
23. State and prove Fatou's Lemma.

24. If (f_n) is a monotone increasing sequence of functions in $M^+(X, X)$ which converges to f, then prove that $\int f\, d\mu = \lim \int f_n\, d\mu$.
25. Suppose that f belongs to M^+. Then prove that $f(x) = 0$ μ–almost everywhere on X if and only if $\int f\, d\mu = 0$.
26. Prove that a measurable function f belongs to L if and only if $|f|$ belongs to L. Also prove that $|\int f\, d\mu| \leq \int |f|\, d\mu$.

SECTION – D

Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks. **(2×6=12)**

27. Prove that f is measurable if and only if the positive and negative part of f is measurable.
28. Prove that the union of a finite collection of measurable sets is measurable.
29. If φ and Ψ are simple functions in $M^+(X, X)$, then prove that $\int(\varphi + \Psi)d\mu = \int \varphi\, d\mu + \int \Psi\, d\mu$.
30. Prove that a constant multiple of αf and a sum f + g of functions in L belongs to L and
 a) $\int \alpha f\, d\mu = \alpha \int f\, d\mu$
 b) $\int (f + g)\, d\mu = \int f\, d\mu + \int g\, d\mu$.

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