K25U 0330

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Mathematics (Honours) Degree (C.B.C.S.S. – Supplementary) Examination, April 2025
(2019 – 2020 Admissions)
Core Course
BHM 603 : OPERATIONS RESEARCH

Time : 3 Hours — Max. Marks : 60

SECTION – A

Answer **any 4** questions out of 5 questions. **Each** question carries **1** mark. **(4×1=4)**

1. Express the following LP problem in equation form :

$$\text{Maximize } z = 2x_1 + 3x_2$$
$$\text{Subject to : } x_1 + 3x_2 \le 6$$
$$3x_1 + 2x_2 \le 6$$
$$x_1, x_2 \ge 0.$$

2. What is meant by an unbounded solution in an LP model ?

3. Determine whether the following transportation model is balanced or not.
Supply : $a_1 = 10, a_2 = 5, a_3 = 4, a_4 = 6$
Demand : $b_1 = 10, b_2 = 5, b_3 = 7, b_4 = 9$.

4. Define a network.

5. What is cut and cut capacity in a network ?

SECTION – B

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks. **(6×2=12)**

6. Solve the LPP graphically :

$$\text{Maximize } z = x_1 + 3x_2$$
$$\text{Subject to : } x_1 + x_2 \le 2$$
$$-x_1 + x_2 \le 4$$
$$x_1 \text{ unrestricted}, x_2 \ge 0.$$

P.T.O.

7. Show graphically that the following LP problem temporarily degenerates.

$$\text{Maximize } z = 3x_1 + 2x_2$$
$$\text{Subject to : } 4x_1 - x_2 \le 8$$
$$4x_1 + 3x_2 \le 12$$
$$4x_1 + x_2 \le 8$$
$$x_1, x_2 \ge 0.$$

8. Write the dual of the LP problem :

$$\text{Maximize } z = 5x_1 + 12x_2 + 4x_3$$
$$\text{Subject to : } x_1 + 2x_2 + x_3 \le 10$$
$$2x_1 - x_2 + 3x_3 = 8$$
$$x_1, x_2, x_3 \ge 0.$$

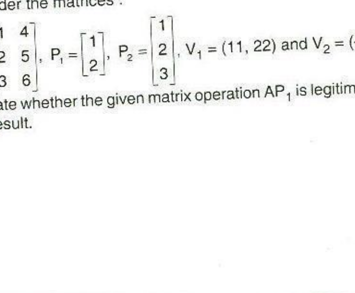
9. Three orchards supply crates of oranges to four retailers. The daily demand amounts at four retailers are 150, 150, 400 and 100 crates respectively. Supplies at the three orchards are dictated by available regular labor and are estimated at 150, 200 and 250 crates daily. However, both orchards 1 and 2 have indicated that they could supply more crates, if necessary, by using overtime labor. Orchard 3 does not offer this option. The transportation costs per crate from the orchards to the retailers are given in the table. Formulate the problem as a transportation model and solve.

		Retailer			
		1	2	3	4
	1	\$1	\$2	\$3	\$4
Orchard	2	\$2	\$4	\$1	\$2
	3	\$1	\$3	\$5	\$3

10. Obtain an initial basic feasible solution to the following transportation problem in which the cells contain the transportation cost in rupees, using north-west corner rule.

	To						
	9	12	9	6	9	10	5
	7	3	7	7	5	5	6
From	6	5	9	11	3	11	2
	6	8	11	2	2	10	9
	4	4	6	2	4	2	22

11. Midwest TV Cable Company is in the process of providing cable service to five new housing development areas. The following figure depicts possible TV linkages among the five areas. The cable miles are shown on each arc. Determine the most economical cable network.



12. Explain Three-Jug Puzzle.

13. Define a project. Explain the role of CPM and PERT in a project.

14. Consider the matrices :

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}, P_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, P_2 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, V_1 = (11, 22) \text{ and } V_2 = (-1, -2, -3).$$

Indicate whether the given matrix operation AP_1 is legitimate and if so, calculate the result.

SECTION – C

Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks. **(8×4=32)**

15. Write the four steps included in the simplex method to solve an LPP.

16. Solve the LPP using M-Method :

$$\text{Maximize } z = 2x_1 + 3x_2 - 5x_3$$
$$\text{Subject to : } x_1 + x_2 + x_3 = 7$$
$$2x_1 - 5x_2 + x_3 \ge 10$$
$$x_1, x_2, x_3 \ge 0.$$

17. Show that the Phase I terminates with two zero artificial variables in the basic solution when solving the following LPP using two phase method.

$$\text{Maximize } z = 3x_1 + 2x_2 + 3x_3$$
$$\text{Subject to : } 2x_1 + x_2 + x_3 = 2$$
$$x_1 + 3x_2 + x_3 = 6$$
$$3x_1 + 4x_2 + 2x_3 = 8$$
$$x_1, x_2, x_3 \ge 0.$$

18. Use the dual problem to show that the basic solution (x_1, x_2) is not optimal for the following LPP.

$$\text{Maximize } z = 2x_1 + 4x_2 + 4x_3 - 3x_4$$
$$\text{Subject to : } x_1 + x_2 + x_3 = 4$$
$$x_1 + 4x_2 + x_4 = 8$$
$$x_1, x_2, x_3, x_4 \ge 0.$$

19. TOYCO assembles three types of toys : trains, trucks and cars using three operations. Available assembly times for the three operations are 430, 460 and 420 minutes per day, respectively and the revenues per toy train, truck and car are \$3, \$2 and \$5 respectively. The assembly times per train for the three operations are 1, 3 and 1 respectively. The corresponding times per truck and per car are (2, 0, 4) and (1, 2, 0) minutes.

a) Write the associated LP model and its dual.

b) Suppose that for toy trains the per-unit time of operation 2 can be reduced from 3 minutes to at most 1.25 minutes. By how much must the per-unit time of operation 1 be reduced to make toy trains just profitable ? The optimal dual prices for the original problem are $y_1 = 1, y_2 = 2$ and $y_3 = 0$.

20. Explain post optimal analysis and what are the changes affecting optimality ?

21. The demand for a perishable item over the next four months is 400, 300, 420 and 380 tons, respectively. The supply capacities for the same months are 500, 600, 200 and 300 tons. The purchase price per ton varies from month to month and is estimated at \$100, \$140, \$120 and \$150 respectively. Because the item is perishable, a current month's supply must be consumed within 3 months (starting with current month). The storage cost per ton per month is \$3. The nature of the item does not allow back-ordering. Solve the problem as a transportation model and determine the optimum delivery schedule for the item over the next four months.

22. Explain the assignment model and Hungarian method to solve it.

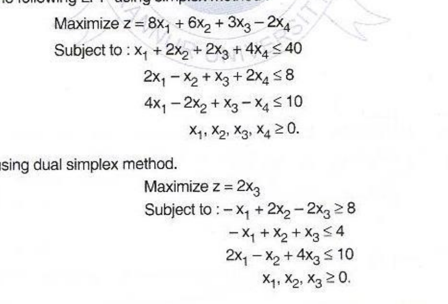
23. Consider the transportation problem in which two factories supply three stores with a commodity. The numbers of supply units available at sources 1 and 2 are 200 and 300; those demanded at stores 1, 2 and 3 are 100, 200 and 50, respectively. Units may be transshipped among the factories and the stores reaching their final destination. Find the optimal shipping schedule based on the unit costs in the following table.

		Factory		Store		
		1	2	1	2	3
	1	\$0	\$6	\$7	\$8	\$9
Factory	2	\$6	\$0	\$5	\$4	\$3
	1	\$7	\$2	\$0	\$5	\$1
Store	2	\$1	\$5	\$1	\$0	\$4
	3	\$8	\$9	\$7	\$6	\$0

24. Write Dijkstra's Algorithm to find shortest route between nodes in a network.

25. Describe the rules for constructing network for a project.

26. Determine the critical path for the project network in the following figure.



SECTION – D

Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks. **(2×6=12)**

27. Solve the following LPP using simplex method :

$$\text{Maximize } z = 8x_1 + 6x_2 + 3x_3 - 2x_4$$
$$\text{Subject to : } x_1 + 2x_2 + 2x_3 + 4x_4 \le 40$$
$$2x_1 - x_2 + x_3 + 2x_4 \le 8$$
$$4x_1 - 2x_2 + x_3 - x_4 \le 10$$
$$x_1, x_2, x_3, x_4 \ge 0.$$

28. Solve using dual simplex method.

$$\text{Maximize } z = 2x_3$$
$$\text{Subject to : } -x_1 + 2x_2 - 2x_3 \ge 8$$
$$-x_1 + x_2 + x_3 \le 4$$
$$2x_1 - x_2 + 4x_3 \le 10$$
$$x_1, x_2, x_3 \ge 0.$$

29. Starting with Vogel's Approximation method find the minimum transportation cost for the following transportation problem.

Source	Destination					Available
	D_1	D_2	D_3	D_4	D_5	
S_1	4	7	3	8	2	4
S_2	1	4	7	3	8	7
S_3	7	2	4	7	7	9
S_4	4	7	2	4	7	2
Required	8	3	7	2	2	

30. Determine the maximal flow in the network shown in the following figure.