K23P 3281

Reg. No. : ....

Name : ....

First Semester M.Sc. Degree (CBSS – Supple. (One Time Mercy Chance)/Imp.) Examination, October 2023

(2014 to 2022 Admissions)

PHYSICS

PHY1C01 : Mathematical Physics – I

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **both** questions (either **a** or **b**), **each** question carries **12** marks.

1. a) Discuss the properties of Pauli matrices. Find the eigenvalues and eigenvectors of Pauli matrices.

OR

b) Show that the eigenvalues of a Hermitian matrix are real and the eigenvectors corresponding to different eigenvalues are orthogonal. Show that the eigenvalues of a unitary matrix is unimodular.

2. a) What is meant by an analytic function ? Obtain the necessary condition for analyticity. Show that the real and imaginary part of a complex function satisfies Laplace equation.

OR

b) Obtain the orthogonality relation of Legendre polynomials. Show that $\int_{-1}^{1} x p_n(x) p_{n-1}(x) dx = \frac{2n}{4n^2 - 1}$.

SECTION – B

Answer **any four** questions, part **a** carries **1** mark, part **b** carries **3** marks and part **c** carries **5** marks.

3. a) A and B are two noncommuting Hermitian matrices. $AB - BA = iC$. Prove that C is Hermitian.

b) Two matrices A and B are each Hermitian. Find the necessary and sufficient condition for their product AB to be Hermitian.

c) Show that $\det e^A = e^{trA}$, where A is an $n \times n$ matrix.

P.T.O.

4. a) Solve Laplace equation $\nabla^2\psi = 0$, in cylindrical coordinates for $\psi = \psi(\rho)$.

b) Obtain the divergence of a vector field in cylindrical polar coordinates.

c) Obtain the curl of a vector field in spherical polar coordinate system.

5. a) What is meant by spectral decomposition of a matrix ?

b) Evaluate $e^{i\sigma_2\theta}$, where σ_2 is the second Pauli matrix.

c) The eigenvectors of a matrix are $\begin{pmatrix}1\\0\\0\end{pmatrix}, \frac{1}{\sqrt{2}}\begin{pmatrix}0\\-1\\1\end{pmatrix}$ and $\frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\1\end{pmatrix}$ with eigenvalues {1/2, 2, 1} respectively. Find the matrix.

6. a) Define metric tensor.

b) Explain what is meant by the rank of a tensor. Discuss the outer product and inner product of tensors.

c) Obtain the components metric tensor g_{ij} and g^{ij} for a sphere of unit radius, S^2.

7. a) Write down the generating function for Bessel function.

b) For integral n, show that $J_{-n}(x) = (-1)^n J_n(x)$.

c) Show that $\frac{d}{dx}\left[x^{-n}J_n(x)\right] = -x^{-n}J_{n+1}(x)$.

8. a) Check the analyticity of the function $f(z) = z^2$.

b) Evaluate the real part of $(i)^i$.

c) Evaluate the integral $\oint_c \frac{dz}{z^2 + z}$, where c is circle defined by with radius $|z| > 1$.
