

Reg. No. : .....

Name : .....

**I Semester M.Sc. Degree (CBSS – Reg./Supple./Imp.)**  
**Examination, October 2021**  
**(2018 Admission Onwards)**  
**PHYSICS**  
**PHY1C01 : Mathematical Physics – I**

Time : 3 Hours

Max. Marks : 60

## SECTION – A

Answer **both** questions, either (a) or (b). Each question carries **12** marks.

1. a) Define eigen values and eigen vectors of a square matrix. Find the eigen values and eigen vectors of the matrix  $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ .

OR

- b) Obtain the series solution to the linear oscillator equation  $y'' + \omega^2 y = 0$  using Frobenius' method.
2. a) Prove that  $\int_0^\infty \frac{\sin x}{x} dx = \frac{\pi}{2}$ .
- OR
- b) Write Laguerre ordinary differential equation and Laguerre polynomial  $L_n(x)$ . Obtain Rodrigues' formula for Laguerre polynomials. Deduce first three Laguerre polynomials. **(2×12=24)**

## SECTION – B

Answer **any four** (1 mark for Part 'a', 3 marks for Part 'b', 5 marks for Part 'c').

3. a) Define divergence of a vector field.
- b) Resolve the circular cylindrical unit vectors into their Cartesian components.
- c) If  $\vec{F} = (x^2 + y^2 + z^2)^n (\hat{e}_x x + \hat{e}_y y + \hat{e}_z z)$ , find the scalar potential of  $\vec{F}$ .

P.T.O.

4. a) Define contravariant tensor.
- b) Explain three dimensional Levi-Civita symbol of tensors.
- c) Define the terms orthogonal matrix, hermitian matrix and unitary matrix. Give examples in each case.
5. a) What do you mean by singular point of an ordinary differential equation ?
- b) What do you mean by Wronskian of an ordinary differential equation ? Discuss the linear independence of solutions of an ordinary differential equation in terms of Wronskian.
- c) Solve the inhomogeneous ordinary differential equation  $(1-x)y'' + xy' - y = (1-x)^2$ .
6. a) Give an example for an analytic function.
- b) Define poles and residues of a complex function. Find the residue of  $f(z) = \frac{1}{\sin z}$  at  $z = 0$ .
- c) State and prove Morera's theorem for a complex function.
7. a) Define beta function.
- b) Prove that  $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$ .
- c) What is the relation between beta and gamma functions ? Prove that  $\int_0^{\frac{\pi}{2}} \cos^{\frac{1}{2}} \theta d\theta = \frac{(2\pi)^{\frac{3}{2}}}{16 \left(\Gamma\left(\frac{5}{4}\right)\right)^2}$ .
8. a) Write the first three Legendre polynomials.
- b) Define spherical Bessel function. Write the expression for  $j_2(x)$ .
- c) For Leguerre polynomials  $L_n(x)$ , prove that  $xL'_n(x) = nL_n(x) - nL_{n-1}(x)$ . **(4×9=36)**