Reg. No. : ....

Name : ....

K22U 1060

II Semester B.Sc. Degree (C.B.C.S.S. – Supplementary) Examination, April 2022
(2016-2018 Admissions)
COMPLEMENTARY COURSE IN MATHEMATICS
2C02 MAT-PH : Mathematics for Physics and Electronics – II

Time : 3 Hours   Max. Marks : 40

SECTION – A

All the first **4** questions are **compulsory**. They carry **1** mark **each**.

1. Give the reduction formula for $\int \cos^n x \, dx$.
2. Write the formula for the area of the surface of the solid obtained on revolving about x – axis, the arc of the curve $y = f(x)$ intercepted between the points whose abscissae are a, b.
3. Two matrices A and B are equal if and only if ______.
4. When is it possible to multiply two matrices of order $m \times n$ and $p \times q$?

SECTION – B

Answer **any 7** questions from among the questions **5** to **13**. These questions carry **2** marks **each**.

5. What is the area of the loop of the curve $r^2 = a^2 \cos 2\theta$?
6. Calculate the area bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
7. Find the whole length of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$.
8. Find the volume of the solid obtained by revolving the circle $x^2 + y^2 = a^2$ about the x – axis.
9. Evaluate $\int_0^1 \int_0^1 \frac{dx\,dy}{\sqrt{1-x^2}\sqrt{1-y^2}}$.

P.T.O.

10. The upper half of the cardioid $r = a(1 + \cos\theta)$ rotates about the polar axis. Find the volume of the solid generated.
11. If A is a 2×2 matrix, show that $A - A^T$ is skew symmetric.
12. Prove that for a 2×2 matrix, the eigenvalues of a diagonal matrix are real.
13. If a square matrix A has its characteristic equation $x^2 - 1 = 0$, prove that its determinant is ± 1.

SECTION – C

Answer **any 4** questions from among the questions **14** to **19**. These questions carry **3** marks **each**.

14. Find the area of one loop of the curve $r = \sqrt{3} \cos 3\theta + \sin 3\theta$.
15. Transform to polar co-ordinates and integrate $\iint \sqrt{\frac{1-x^2-y^2}{1+x^2+y^2}}\, dx\,dy$ the integral being extended over all positive values of x and y subject to $x^2 + y^2 \leq 1$.
16. Solve the system of linear equations
    $x + y + z = 1$
    $2x + 3z = 2$
    $4x + 5y + z = 3$
    by Gaussian Elimination.
17. Verify Cayley-Hamilton theorem for the matrix $A = \begin{pmatrix} 2 & 3 \\ -1 & 5 \end{pmatrix}$.
18. Prove that the eigenvalues of a 3×3 lower triangular matrix are the same as its diagonal entries.
19. Verify that $(A^{-1})^{-1} = A$ for the matrix $A = \begin{pmatrix} 5 & -3 \\ 1 & 1 \end{pmatrix}$.

SECTION – D

Answer **any 2** questions from among the questions **20** to **23**. These questions carry **5** marks **each**.

20. Find the intrinsic equation of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$ when s is measured from
    i) the vertex
    ii) the cusp on the x – axis.

21. Show that if the area lying within the cardioid $r = 2a(1 + \cos\theta)$ and without the parabola $r(1 + \cos\theta) = 2a$ revolves about the initial line, the volume generated is $18\pi a^3$.
22. Find the eigenvalues and eigenvectors for $A = \begin{pmatrix} 10 & 2 \\ 3 & 5 \end{pmatrix}$.
23. Consider the system of linear equations
    $x + y = 3$
    $x + 2y + z = 5$
    $x + y + az = b$
    i) For which values of a and b the system has a unique solution ? Why ?
    ii) For which values of a and b the system is inconsistent ? Why ?

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