Reg. No. : ....

Name : ....

V Semester B.Sc. Honours in Mathematics Degree
(C.B.C.S.S. – Supplementary) Examination, November 2024
(2019 and 2020 Admissions)
BMH 504 : DIFFERENTIAL GEOMETRY

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any four** questions from the following. **Each** question carries **1** mark. **(4×1=4)**

1. Sketch the vector field $X(p) = -p$ on R^2.
2. Define the graph of a function $f : U \to R$, $U \subseteq R^{n+1}$.
3. Find the regular points of the function $f : R^2 \to R$ defined by $f(x_1, x_2) = x_1^2 - x_2^2$.
4. Give an example of an n-surface in R^{n+1}.
5. Find the speed of the parameterized curve $\alpha(t) = (2 \sin t, 2 \cos t)$.

SECTION – B

Answer **any six** questions. **Each** question carries **2** marks. **(6×2=12)**

6. Sketch the graph of the function $f(x_1, x_2) = x_1^2 - x_2^2$.
7. Show that graph of any function $f : R^n \to R$ is a level set for some function $F : R^{n+1} \to R$.
8. Define a consistent ordered basis for the tangent space S_p.
9. Find $\nabla f(p)$ for $f(x_1, x_2) = 2x_1^2 + 3x_2^2$ at $p = (1, -1)$.
10. Show that the gradient of f at $p \in f^{-1}(c)$ is orthogonal to all vectors tangent to $f^{-1}(c)$ at p.

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11. Show by an example that the set of all vectors tangent at a point p of a level set might be all of R_p^{n+1}.
12. If X and Y are two parallel vector fields along α, show that X.Y is constant along α.
13. With the usual notations, show that $\nabla_{v+w} f = \nabla_v f + \nabla_w f$.
14. Prove that $D_v(X + Y) = D_v X + D_v Y$.

SECTION – C

Answer **any eight** questions. **Each** question carries **4** marks **each**. **(8×4=32)**

15. Find the velocity and acceleration of the parametrized curve $\alpha(t) = (\cos t, \sin t, 2\cos t, 2\sin t)$.
16. Prove that $x_1^2 + 3x_2^2 + x_3^2 = 1$ is an 2-surface in R^3.
17. Does the parameterized curve $\alpha(t) = (\cos t, \sin t, t)$ is a geodesic in the cylinder $x_1^2 + x_2^2 = 1$ in R^3. Justify your answer.
18. Compute $\nabla_v f$ where $f(x_1, x_2) = x_1^3 + x_2^3 - 3x_1x_2^2$, $v = (-1, 1, -1, -1)$.
19. Let S be an n-surface in R^{n+1} and let, $\alpha : I \to S$ be a parametrized curve and let X be a vector field tangent to S along α. Prove that $(fX)' = f'X + fX'$.
20. Find the integral curve through $(-1, 1)$ of the vector field $X(x_1, x_2) = (x_1, x_2, -x_2, -x_1)$.
21. Prove that an n-sphere is connected.
22. Sketch the cylinder over the graph of $f(x) = \sin 2x$.
23. State and prove Lagrange Multiplier Theorem.
24. Show that the two orientations on the n-sphere $x_1^2 + x_2^2 + \ldots + x_{n+1}^2 = 1$ given by $N_1(p) = (p, p)$ and $N_2(p) = (p, -p)$.
25. Sketch the surface of revolution obtained by rotating the curve $x_2 = x_1$, $0 < x_1 < 1$ about the x_1-axis.
26. Compute the Weingarten map for the cylinder $x_2^2 + x_3^2 = 1$ in R^3.

SECTION – D

Answer **any two** questions. **Each** question carries **6** marks **each**. **(2×6=12)**

27. Let $S \subset R^{n+1}$ b a connected n surface in R^{n+1}. Prove that there exist on S exactly two orientations N_1 and N_2, $N_2(p) = -N_1(p)$ for all p in S.
28. Sketch the level curves $(c = -2, 0, 2)$ and graph of the function $f(x_1, x_2) = x_1^2 - x_2^2$.
29. Show that the set S of all unit vectors at all points of R^2 forms a 3-surface in R^4.
30. With the usual notations, prove that the parallel transport $P_\alpha : S_p \to S_q$ along α is a vector space isomorphism which preserves dot product.

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