K24U 3162

Reg. No. : ....

Name : ....

V Semester B.Sc. Honours in Mathematics Degree (C.B.C.S.S. – Supplementary) Examination, November 2024
(2019 and 2020 Admissions)
BHM 502 : ADVANCED COMPLEX ANALYSIS

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any 4** questions out of 5 questions. **Each** question carries **1** mark. **(4×1=4)**

1. Evaluate $\int_0^{\pi/4} e^{it}dt$.

2. State Cauchy Goursat theorem.

3. Let $z_n = -2 + i\frac{(-1)^n}{n^2}$, n = 1, 2, . . . Find $\lim_{n\to\infty} z_n$.

4. Identify the singularity of $f(z) = \frac{z^2 - 2z + 3}{z - 2}$ at z = 2.

5. Define meromorphic function.

SECTION – B

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks. **(6×2=12)**

6. By finding an antiderivative, evaluate $\int_1^3 (z-2)^3 dz$.

7. State Cauchy integral formula and use this formula to evaluate $\int_C \frac{z\,dz}{(9-z^2)(z+i)}$, where C is the positively oriented circle |z| = 2.

P.T.O.

8. Let C denote a contour of length L, and suppose that a function f(z) is piecewise continuous on C. If M is a nonnegative constant such that $|f(z)| \le M$ for all points z on C at which f(z) is defined, prove that $\left|\int_C f(z)\,dz\right| \le ML$.

9. If a series of complex numbers converges, prove that n^{th} term converges to zero as $n \to \infty$.

10. Obtain the Taylor series for the function $f(z) = e^z$ about the point $z_0 = 1$.

11. State Laurent's theorem.

12. Use residue to evaluate $\int_C \frac{dz}{z(z-2)^4}$, where C is the positively oriented circle |z – 2| = 1.

13. Show that $\operatorname{Res}_{z=i} \frac{\log z}{(z^2+1)^2} = \frac{\pi + 2i}{8}$.

14. Determine the number of zeros, counting multiplicities of the polynomial $z^4 + 3z^3 + 6$ inside the circle |z| = 2.

SECTION – C

Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks. **(8×4=32)**

15. Use Parametric representation for C, or legs of C, to evaluate $\int_C f(z)dz$, where $f(z) = \pi\exp(\pi\bar{z})$ and C is the boundary of the square with vertices at the points 0, 1, 1 + i and i the orientation of C being in the counter clockwise direction.

16. Without evaluating the integral, show that $\left|\int_C \frac{dz}{z^2-1}\right| \le \frac{\pi}{3}$, where C is the arc of the circle |z| = 2 from z = 2 to z = 2i.

17. State and prove Liouville's theorem.

18. Represent the function $f(z) = \frac{-1}{(z-1)(z-2)}$
    i) by its Maclaurin series and state where the representation is valid.
    ii) by its Laurent series in the domain 1 < |z| < 2.

19. Suppose z_1 is a point inside the circle of convergence $|z - z_0| = R$ of a power series $\sum_{n=0}^{\infty} a_n(z-z_0)^n$. Prove that the series is uniformly convergent in the closed disk $|z - z_0| < R_1$, where $R_1 = |z_1 - z_0|$.

20. Find the Taylor series for the function $\frac{1}{z}$ about the point $z_0 = 2$. Then by differentiating that series term by term, show that $\frac{1}{z^2} = \frac{1}{4}(n+1)\left(\frac{z-2}{2}\right)^n$, |z – 2|< 2.

21. If a function f is analytic everywhere in the finite plane except for a finite number of singular points interior to a positively oriented simple closed contour C, prove that $\int_C f(z)dz = 2\pi i \operatorname{Res}_{z=0}\left[\frac{1}{z^2} f\left(\frac{1}{z}\right)\right]$.

22. Find the value of the integral $\int_C \frac{dz}{z^3(z+4)}$ taken counter clockwise around the circle |z| = 2.

23. Let f be a function analytic at a point z_0. Prove that f has a zero of order m at z_0 if and only if there is a function g, which is analytic and nonzero at z_0 such that $f(z) = (z - z_0)^m g(z)$.

24. Evaluate $\int_0^{\infty} \frac{x^2}{x^6+1}dx$.

25. Find Cauchy principal value of the improper integral $\int_{-\infty}^{\infty} \frac{\sin x\,dx}{x^2+4x+5}$.

26. State and prove Rouche's theorem.

SECTION – D

Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks. **(2×6=12)**

27. State and prove fundamental theorem of algebra.

28. Suppose that a function f is analytic throughout a disk $|z - z_0| < R_0$, centered at z_0 and with radius R_0. Then prove that f(z) has the power series representation $f(z) = \sum_{n=0}^{\infty} a_n(z-z_0)^n, |z - z_0| < R_0$ where $a_n = \frac{f^{(n)}(z_0)}{n!}$.

29. State and prove Cauchy residue theorem.

30. Derive the integral formula $\int_{-\infty}^{\infty} \frac{\cos ax - \cos bx}{x^2}dx = \frac{\pi}{2}(b-a)$, a ≥ 0, b ≥ 0.

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