



K24U 3162

Reg. No. : .....

Name : .....

**V Semester B.Sc. Honours in Mathematics Degree (C.B.C.S.S. –  
Supplementary) Examination, November 2024  
(2019 and 2020 Admissions)  
BHM 502 : ADVANCED COMPLEX ANALYSIS**

Time : 3 Hours

Max. Marks : 60

**SECTION – A**Answer **any 4** questions out of 5 questions. **Each** question carries **1** mark. **(4×1=4)**

1. Evaluate  $\int_0^{\pi/2} e^{it} dt$ .
2. State Cauchy Goursat theorem.
3. Let  $z_n = -2 + i \frac{(-1)^n}{n^2}$ ,  $n = 1, 2, \dots$ . Find  $\lim_{n \rightarrow \infty} z_n$ .
4. Identify the singularity of  $f(z) = \frac{z^2 - 2z + 3}{z - 2}$  at  $z = 2$ .
5. Define meromorphic function.

**SECTION – B**Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks. **(6×2=12)**

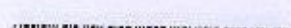
6. By finding an antiderivative, evaluate  $\int_1^3 (z - 2)^3 dz$ .
7. State Cauchy integral formula and use this formula to evaluate  $\int_C \frac{z dz}{(9 - z^2)(z + i)}$

where C is the positively oriented circle  $|z| = 2$ .

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8. Let C denote a contour of length L, and suppose that a function  $f(z)$  is piecewise continuous on C. If M is a nonnegative constant such that  $|f(z)| \leq M$  for all points z on C at which  $f(z)$  is defined, prove that  $\left| \int_C f(z) dz \right| \leq ML$ .
9. If a series of complex numbers converges, prove that  $n^{\text{th}}$  term converges to zero as  $n \rightarrow \infty$ .
10. Obtain the Taylor series for the function  $f(z) = e^z$  about the point  $z_0 = 1$ .
11. State Laurent's theorem.
12. Use residue to evaluate  $\int_C \frac{dz}{z(z-2)^4}$ , where C is the positively oriented circle  $|z - 2| = 1$ .
13. Show that  $\text{Res}_{z=i} \frac{\log z}{(z^2 + 1)^2} = \frac{\pi + 2i}{8}$ .
14. Determine the number of zeros, counting multiplicities of the polynomial  $z^4 + 3z^3 + 6$  inside the circle  $|z| = 2$ .

**SECTION – C**Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks. **(8×4=32)**

15. Use Parametric representation for C, or legs of C, to evaluate  $\int_C f(z) dz$ , where  $f(z) = \pi \exp(\pi \bar{z})$  and C is the boundary of the square with vertices at the points 0, 1,  $1 + i$  and  $i$  the orientation of C being in the counter clockwise direction.
16. Without evaluating the integral, show that  $\left| \int_C \frac{dz}{z^2 - 1} \right| \leq \frac{\pi}{3}$ , where C is the arc of the circle  $|z| = 2$  from  $z = 2$  to  $z = 2i$ .
17. State and prove Liouville's theorem.
18. Represent the function  $f(z) = \frac{-1}{(z-1)(z-2)}$ 
  - i) by its Maclaurin series and state where the representation is valid.
  - ii) by its Laurent series in the domain  $1 < |z| < 2$ .



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19. Suppose  $z_1$  is a point inside the circle of convergence  $|z - z_0| = R$  of a power series  $\sum_{n=0}^{\infty} a_n (z - z_0)^n$ . Prove that the series is uniformly convergent in the closed disk  $|z - z_0| < R_1$ , where  $R_1 = |z_1 - z_0|$ .
20. Find the Taylor series for the function  $\frac{1}{z}$  about the point  $z_0 = 2$ . Then by differentiating that series term by term, show that  $\frac{1}{z^2} = \frac{1}{4} (n+1) \left( \frac{z-2}{2} \right)^n$ ,  $|z - 2| < 2$ .
21. If a function  $f$  is analytic everywhere in the finite plane except for a finite number of singular points interior to a positively oriented simple closed contour C, prove that  $\int_C f(z) dz = 2\pi i \text{Res}_{z=0} \left[ \frac{1}{z^2} f\left(\frac{1}{z}\right) \right]$ .
22. Find the value of the integral  $\int_C \frac{dz}{z^3(z+4)}$  taken counter clockwise around the circle  $|z| = 2$ .
23. Let  $f$  be a function analytic at a point  $z_0$ . Prove that  $f$  has a zero of order  $m$  at  $z_0$  if and only if there is a function  $g$ , which is analytic and nonzero at  $z_0$  such that  $f(z) = (z - z_0)^m g(z)$ .
24. Evaluate  $\int_0^{\infty} \frac{x^2}{x^6 + 1} dx$ .
25. Find Cauchy principal value of the improper integral  $\int_{-\infty}^{\infty} \frac{\sin x dx}{x^2 + 4x + 5}$ .
26. State and prove Rouché's theorem.

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**SECTION – D**Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks. **(2×6=12)**

27. State and prove fundamental theorem of algebra.
28. Suppose that a function  $f$  is analytic throughout a disk  $|z - z_0| < R_0$ , centered at  $z_0$  and with radius  $R_0$ . Then prove that  $f(z)$  has the power series representation  $f(z) = \sum_{n=0}^{\infty} a_n (z - z_0)^n$ ,  $|z - z_0| < R_0$  where  $a_n = \frac{f^{(n)}(z_0)}{n!}$ .
29. State and prove Cauchy residue theorem.

30. Derive the integral formula  $\int_{-\infty}^{\infty} \frac{\cos ax - \cos bx}{x^2} dx = \frac{\pi}{2} (b - a)$ ,  $a \geq 0$ ,  $b \geq 0$ .