Name :

Sixth Semester B.Sc. Mathematics (Honours) Degree (C.B.C.S.S. - OBE - Regular) Examination, April 2024 (2021 Admissions) Core Course

6B25 BMH: TOPOLOGY

Time: 3 Hours

Max. Marks: 60

SECTION - A

Answer any 4 questions out of 5 questions. Each carries 1 mark :

 $(4 \times 1 = 4)$

Give an example of a bounded metric on R.

- 2. The limit point of the set $\{1, \frac{1}{2}, \frac{1}{3}, ...\}$ is
- Write a basis for the discrete topology on a set X.
- Is the set of rational numbers Q connected ? Justify. Give an example of an Hausdorff space which is not regular.
- SECTION B

Answer any 6 questions out of 9 questions. Each carries 2 marks : 6. If (X, d) is a metric space, then show that d, defined by d, (x, y) = $\frac{d(x, y)}{1 + d(x, y)}$

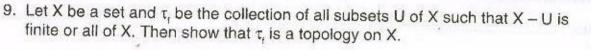
 $(6 \times 2 = 12)$

- is also a metric on X. 7. Define open spheres on a metric space (X, d). What are the open spheres on
- Let X be a complete metric space and Y ⊆ X. Then show that Y is complete iff Y is closed in X.

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the real line?



- 10. Show that the function $f: \mathbb{R} \to \mathbb{R}$ given by f(x) = 3x + 1 is a homcomorphism and find its inverse.
- 11. Prove that a space X is connected iff the only subsets of X that are both open and closed in X are the empty set and X itself.
- 12. Show that R is not compact.
- 13. Show by an example that the product of two Lindelof spaces need not be Lindelof.
- 14. Show that the space \mathbb{R}_i is normal.
- SECTION C Answer any 8 questions out of 12 questions. Each carries 4 marks :

15. Let X be a metric space. Show that a subset G of X is open iff it is a union of open spheres. Show that the Cantor set is uncountable.

on R but are not comparable with one another.

 $(8 \times 4 = 32)$

- 17. Let X and Y be metric spaces and $f: X \rightarrow Y$. Then show that f is continuous iff $x_n \to x$ implies $f(x_n) \to f(x)$. 18. Show that the topologies \mathbb{R} , and \mathbb{R}_{κ} are strictly finer than the standard topology
- points of A. Then show that $A = A \cup A'$. State and prove pasting lemma.

19. Let A be a subset of the topological space X. Let A' be the subset of all limit

- 21. Let A be a connected subspace of X. If $A \subset B \subset \overline{A}$, then prove that B is also
- 22. Prove that the image of a connected space under a continuous map is connected. State and prove intermediate value theorem.

24. Show that \mathbb{R}_i is not second countable.

Hausdorff spaces is Hausdorff.

26. Show that every compact Hausdorff space is normal.

SECTION - D

 $(2 \times 6 = 12)$

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disjoint class of open intervals. Prove the following about Hausdorff spaces.

a) Every finite point set in a Hausdorff space is closed.

Answer any 2 questions out of 4 questions. Each carries 6 marks :

b) If X is a Hausdorff space then a sequence of points of X converges to atmost one point of X.

29. Let A be an open covering of the metric space (X, d). If X is compact then show that there is a δ < 0 such that for each subset of X having diameter less than δ

27. Prove that every non empty open set on the real line is the union of a countable

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25. Prove that a subspace of Hausdorff space is Hausdorff and product of

- there exist an element of A containing it. Prove that every regular space with a countable basis is normal.