Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Mathematics (Honours) Degree (C.B.C.S.S. – OBE – Regular) Examination, April 2024
(2021 Admissions)
Core Course
6B25 BMH : TOPOLOGY

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any 4** questions out of 5 questions. **Each** carries **1** mark : **(4×1=4)**

1. Give an example of a bounded metric on $\mathbb{R}$.
2. The limit point of the set $\left\{1, \frac{1}{2}, \frac{1}{3}, ...\right\}$ is
3. Write a basis for the discrete topology on a set X.
4. Is the set of rational numbers $\mathbb{Q}$ connected ? Justify.
5. Give an example of an Hausdorff space which is not regular.

SECTION – B

Answer **any 6** questions out of 9 questions. **Each** carries **2** marks : **(6×2=12)**

6. If (X, d) is a metric space, then show that d_1 defined by $d_1(x, y) = \frac{d(x, y)}{1+d(x, y)}$ is also a metric on X.
7. Define open spheres on a metric space (X, d). What are the open spheres on the real line ?
8. Let X be a complete metric space and $Y \subseteq X$. Then show that Y is complete iff Y is closed in X.

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9. Let X be a set and τ_f be the collection of all subsets U of X such that X – U is finite or all of X. Then show that τ_f is a topology on X.
10. Show that the function $f : \mathbb{R} \to \mathbb{R}$ given by f(x) = 3x + 1 is a homcomorphism and find its inverse.
11. Prove that a space X is connected iff the only subsets of X that are both open and closed in X are the empty set and X itself.
12. Show that $\mathbb{R}$ is not compact.
13. Show by an example that the product of two Lindelof spaces need not be Lindelof.
14. Show that the space $\mathbb{R}_l$ is normal.

SECTION – C

Answer **any 8** questions out of 12 questions. **Each** carries **4** marks : **(8×4=32)**

15. Let X be a metric space. Show that a subset G of X is open iff it is a union of open spheres.
16. Show that the Cantor set is uncountable.
17. Let X and Y be metric spaces and $f : X \to Y$. Then show that f is continuous iff $x_n \to x$ implies $f(x_n) \to f(x)$.
18. Show that the topologies $\mathbb{R}_l$ and $\mathbb{R}_K$ are strictly finer than the standard topology on $\mathbb{R}$ but are not comparable with one another.
19. Let A be a subset of the topological space X. Let A′ be the subset of all limit points of A. Then show that $\overline{A} = A \cup A'$.
20. State and prove pasting lemma.
21. Let A be a connected subspace of X. If $A \subset B \subset \overline{A}$, then prove that B is also connected.
22. Prove that the image of a connected space under a continuous map is connected.
23. State and prove intermediate value theorem.

24. Show that $\mathbb{R}_l$ is not second countable.
25. Prove that a subspace of Hausdorff space is Hausdorff and product of Hausdorff spaces is Hausdorff.
26. Show that every compact Hausdorff space is normal.

SECTION – D

Answer **any 2** questions out of 4 questions. **Each** carries **6** marks : **(2×6=12)**

27. Prove that every non empty open set on the real line is the union of a countable disjoint class of open intervals.
28. Prove the following about Hausdorff spaces.
    a) Every finite point set in a Hausdorff space is closed.
    b) If X is a Hausdorff space then a sequence of points of X converges to atmost one point of X.
29. Let A be an open covering of the metric space (X, d). If X is compact then show that there is a $\delta < 0$ such that for each subset of X having diameter less than δ there exist an element of A containing it.
30. Prove that every regular space with a countable basis is normal.

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