Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Honours in Mathematics Degree
(CBCSS – Improvement/Supplementary/One Time Mercy Chance)
Examination, April 2024
(2016 to 2020 Admissions)
Core Course
BHM 602 : TOPOLOGY

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any four** questions out of the five questions. **Each** question carries **1** mark. **(4×1=4)**

1. Give an example of a non-hereditary property.
2. Give an example of a nearness relation.
3. When is a subset of a space said to be Lindeloff ?
4. What is a Lebesgue number ?
5. Give an example of a Hausdorff space which is not metrisable.

SECTION – B

Answer **any six** questions out of the nine questions. **Each** question carries **2** marks. **(6×2=12)**

1. Give an example of a base for discrete topology in a set X. Further, explain how it forms a base for the topology.
2. Let (X, d) be a metric space. Given two distinct points $x, y \in X$, show that there exist disjoint open sets U, V such that $x \in U$ and $y \in V$.
3. Let (X, τ) be a second countable space and $Y \subset X$. Prove that any cover of Y by members of τ has a countable subcover.

4. Is the identity function always continuous ? Justify your answer.
5. Prove that for a function $f : X \to Y$ continuous at x_0 the inverse image of every neighbourhood of $f(x_0)$ in Y is a neighbourhood of x_0 in X.
6. Show that $\mathbb{Q}$ the set of rational numbers is disconnected in $\mathbb{R}$ with usual topology.
7. Let X_1, X_2 be connected topological spaces. Prove that $X = X_1 \times X_2$, with the product topology, is connected.
8. Prove that regularity is a hereditary property.
9. State Urysohn's Lemma.

SECTION – C

Answer **any eight** questions out of the twelve questions. **Each** question carries **4** marks. **(8×4=32)**

1. Define cofinite topology on a set X. Show that it is a topology on X.
2. Let X be a set and $\mathcal{B}$ a family of subsets covering X. If for any $B_1, B_2 \in \mathcal{B}$ and $x \in B_1 \cap B_2$ there exists $B_3 \in \mathcal{B}$ such that $x \in B_3 \subset B_1 \cap B_2$, then prove that $\mathcal{B}$ is a base for some topology on X.
3. Prove that a discrete space is second countable if and only if the underlying set is countable.
4. Prove that $\bar{A} = A \cup A'$.
5. Let (X, τ), (Y, u) be spaces and $f : X \to Y$ be function. Prove that f is continuous if and only if for every $V \in u$ we have $f^{-1}(V) \in \tau$.
6. Prove that $\bar{A} = \{y \in X$: every neighbourhood of y meets A non-vacuously$\}$.
7. Prove that every continuous real valued function on a compact space is bounded and attains its extrema.
8. Prove that every separable space satisfies the countable chain condition.
9. Define mutually separated sets. Let $\mathcal{C}$ be a collection of connected subsets of X no two of which are mutually separated. Prove that $\bigcup_{C \in \mathcal{C}} C$ is connected.

10. Prove that every regular, Lindeloff space is normal.
11. Define an accumulation point. If y is an accumulation point of a subset A of a T_1 space X, prove that every neighbourhood of y contains infinitely many points of A.
12. Give an example, with proper justification, of a space which is Hausdorff but not regular.

SECTION – D

Answer **any two** questions out of the four questions. **Each** question carries **6** marks. **(2×6=12)**

1. a) Define semi-open interval topology on $\mathbb{R}$. Show that it is stronger than the usual topology on $\mathbb{R}$.

   b) Let (X, τ) be a space. If S is a sub-base for τ prove that τ is the smallest topology on X containing S.

2. a) Is $\bar{A} \cup \bar{B} = \overline{A \cup B}$? Justify your answer.

   b) Let X be the topological product of the topological spaces $\{(X_i, \tau_i) : i = 1, 2, ..., n\}$. If Z is any space, then prove that $f : Z \to X$ is continuous if and only if $\pi_i \circ f : Z \to X_i$ is continuous for all i = 1, 2, ..., n.

3. For $\mathbb{R}$ with usual topology, prove that a subset of $\mathbb{R}$ is connected if and only if it is an interval.

4. a) Prove that all metric spaces are T_4.

   b) Prove that a compact subset of a Hausdorff space is closed.

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