Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Honours in Mathematics Degree (C.B.C.S.S. – Supplementary/Improvement/One Time Mercy Chance) Examination, April 2024
(2016 to 2020 Admissions)
Core Course
BHM 601 : MATHEMATICAL TRANSFORMS

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any 4** questions out of 5 questions. **Each** question carries **1** mark. **(4×1=4)**

1. Find the Laplace transform of the function, $f(t) = e^{2t}$, for $t > 0$.
2. If $\mathscr{L}\{f(t)\} = F(s)$, then $\mathscr{L}^{-1}\{e^{-as} F(s)\}$ is
3. Find the Fourier sine transform of the function, $f(x) = \begin{cases} k, & \text{if } 0 < x < a \\ 0, & \text{if } x > a \end{cases}$.
4. Write the zero order Hankel transforms of $\frac{\delta(r)}{r}$.
5. Find the inverse Z transform of $\frac{z}{(z-a)^2}$.

SECTION – B

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks. **(6×2=12)**

6. Find the inverse Laplace transform of $\frac{2s+16}{s^2-16}$.
7. Write the Laplace transform of $f(t) = \sin\omega t$.
8. Find the Fourier cosine transform $\mathscr{F}_c(f(x))$ if $f(x) = e^{-ax}$, where $a > 0$.
9. Find the first order Hankel transforms of $f(r) = \frac{\sin ar}{r}$.
10. Find the Mellin transform of the function $f(x) = (e^x - 1)^{-1}$.

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11. Find Z {cosh nx}.
12. Find the inverse Z transform of $F(z) = \frac{z}{z-a}$.
13. If $Z\{f(n)\} = F(z)$, find $Z\{e^{-nb} f(n)\}$.
14. State and prove the shifting property of Mellin transformation.

SECTION – C

Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks. **(8×4=32)**

15. Solve the initial value problem $y'' + 2y' + 2y = 0$, $y(0) = 1$, $y'(0) = -3$.
16. Write the function f(t) using unit step function and find its Laplace transform if
$$f(t) = \begin{cases} 2, & \text{if } 0 < 1 \\ \frac{1}{2}t^2, & \text{if } 1 < t < \frac{\pi}{2} \\ \cos t, & \text{if } \frac{\pi}{2} < t \end{cases}.$$
17. State and prove convolution theorem for Laplace transforms.
18. Show that $\int_0^\infty \frac{\cos x\omega + \omega \sin x\omega}{1+\omega^2} d\omega = \begin{cases} 0, & \text{if } x < 0 \\ \frac{\pi}{2}, & \text{if } x = 0 \\ \pi e^{-x}, & \text{if } x > 0 \end{cases}$.
19. Prove that $\mathscr{F}_c(f'(x)) = \omega \mathscr{F}_s(f(x)) - \sqrt{\frac{2}{\pi}} f(0)$.
20. Let f(x) be continuous on the x-axis and $f(x) \to 0$ as $|x| \to \infty$ and let $f'(x)$ be absolutely integrable on the x-axis. Then prove that $\mathscr{F}[f'(x)] = i\omega \mathscr{F}\{f(x)\}$.
21. If $\tilde{f}_n(k) = \mathscr{H}\{f(n)\}$, prove that $\mathscr{H}_n\{f'(r)\} = \frac{k}{2n}\left[(n-1)\tilde{f}_{n-1}(k) - (n+1)\tilde{f}_{n+1}(k)\right]$, $n \geq 1$.
22. Prove that $\mathscr{M}\left\{\int_0^x f(t)\,dt\right\} = -\frac{1}{p}\tilde{f}(p+1)$.

23. Prove that $\mathscr{M}\left(\frac{1}{e^x + e^{-x}}\right) = \Gamma(p).L(p)$ where $L(p) = \sum_{n=0}^{\infty} (-1)^n \frac{1}{(2n+1)^p}$.
24. Find the inverse Z transform of $\frac{z^2}{(z-a)(z-b)}$.
25. If $Z\{f(n)\} = F(z)$ and $Z\{g(n)\} = G(z)$, then prove that the Z transform of the convolution $f(n) * g(n)$ is given by $Z\{f(n) * g(n)\} = Z\{f(n)\}Z\{g(n)\}$, where $f(n) * g(n) = \sum_{m=0}^{n} f(n-m)g(m)$.
26. Using Z transform, find $\sum_{n=0}^{\infty} \frac{(-1)^n e^{-n}}{n+1}$.

SECTION – D

Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks. **(2×6=12)**

27. Solve the initial value problem :
$y_1' = -y_1 - y_2$, $y_2' = y_1 - y_2$
$y_1(0) = 0$, $y_2(0) = 1$.
28. Suppose that f(x) and g(x) are piecewise continuous, bounded and absolutely integrable on the x-axis. Then prove that $\mathscr{F}(f^*g) = \sqrt{2\pi}\,\mathscr{F}(f).\mathscr{F}(g)$.
29. If $\tilde{f}_n(k) = \mathscr{H}\{f(n)\}$, prove that
$$\mathscr{H}_n\left\{\left(\nabla^2 - \frac{n^2}{r^2}\right)f(r)\right\} = \mathscr{H}_n\left\{\frac{1}{r}\frac{d}{dr}\left(r\frac{df}{dr}\right) - \frac{n^2}{r^2}f(r)\right\} = -K^2\tilde{f}_n(K)$$
provided both $rf'(r)$ and $rf(r)$ vanish as $r \to 0$ and $r \to \infty$.
30. Solve : $u(n+2) - u(n+1) + u(n) = 0$, $u(n) = 0$, $u(1) = 2$.

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