

Reg. No. :

Name :

**Sixth Semester B.Sc. Honours in Mathematics Degree (C.B.C.S.S. –
Supplementary/Improvement/One Time Mercy Chance) Examination, April 2024
(2016 to 2020 Admissions)**

Core Course**BHM 601 : MATHEMATICAL TRANSFORMS**

Time : 3 Hours

Max. Marks : 60

SECTION – A

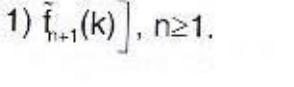
Answer any 4 questions out of 5 questions. Each question carries 1 mark. (4x1=4)

1. Find the Laplace transform of the function, $f(t) = e^{2t}$, for $t > 0$.2. If $\mathcal{L}\{f(t)\} = F(s)$, then $\mathcal{L}^{-1}\{e^{-as} F(s)\}$ is3. Find the Fourier sine transform of the function, $f(x) = \begin{cases} k, & \text{if } 0 < x < a \\ 0, & \text{if } x > a \end{cases}$.4. Write the zero order Hankel transforms of $\frac{\delta(r)}{r}$.5. Find the inverse Z transform of $\frac{z}{(z-a)^2}$.**SECTION – B**

Answer any 6 questions out of 9 questions. Each question carries 2 marks. (6x2=12)

6. Find the inverse Laplace transform of $\frac{2s+16}{s^2-16}$.7. Write the Laplace transform of $f(t) = \sin \omega t$.8. Find the Fourier cosine transform $\mathcal{F}(f(x))$ if $f(x) = e^{-ax}$, where $a > 0$.9. Find the first order Hankel transforms of $f(r) = \frac{\sin ar}{r}$.10. Find the Mellin transform of the function $f(x) = (e^x - 1)^{-1}$.

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11. Find $Z\{\cosh nx\}$.12. Find the inverse Z transform of $F(z) = \frac{z}{z-a}$.13. If $Z\{f(n)\} = F(z)$, find $Z\{e^{-nb} f(n)\}$.

14. State and prove the shifting property of Mellin transformation.

SECTION – C

Answer any 8 questions out of 12 questions. Each question carries 4 marks. (8x4=32)

15. Solve the initial value problem $y'' + 2y' + 2y = 0$, $y(0) = 1$, $y'(0) = -3$.16. Write the function $f(t)$ using unit step function and find its Laplace transform if

$$f(t) = \begin{cases} 2, & \text{if } 0 < t \\ \frac{1}{2}t^2, & \text{if } 1 < t < \frac{\pi}{2} \\ \cos t, & \text{if } \frac{\pi}{2} < t \end{cases}$$

17. State and prove convolution theorem for Laplace transforms.

18. Show that $\int_0^\infty \frac{\cos x\omega + \omega \sin x\omega}{1+\omega^2} d\omega = \begin{cases} 0, & \text{if } x < 0 \\ \frac{\pi}{2}, & \text{if } x = 0 \\ \pi e^{-x}, & \text{if } x > 0 \end{cases}$.19. Prove that $\mathcal{F}_c(f'(x)) = \omega \mathcal{F}_s(f(x)) - \sqrt{\frac{2}{\pi}} f(0)$.20. Let $f(x)$ be continuous on the x -axis and $f(x) \rightarrow 0$ as $|x| \rightarrow \infty$ and let $f'(x)$ be absolutely integrable on the x -axis. Then prove that $\mathcal{F}[f'(x)] = i\omega \mathcal{F}\{f(x)\}$.21. If $\tilde{f}_n(k) = \mathcal{H}\{f(n)\}$, prove that $\mathcal{H}_n\{f'(r)\} = \frac{k}{2n} [(n-1)\tilde{f}_{n-1}(k) - (n+1)\tilde{f}_{n+1}(k)]$, $n \geq 1$.22. Prove that $\mathcal{M}\left\{\int_0^x f(t) dt\right\} = -\frac{1}{p} \tilde{f}(p+1)$.**SECTION – D**

Answer any 2 questions out of 4 questions. Each question carries 6 marks. (2x6=12)

27. Solve the initial value problem :

$$y'_1 = -y_1 - y_2, \quad y'_2 = y_1 - y_2$$

$$y_1(0) = 0, \quad y_2(0) = 1$$

28. Suppose that $f(x)$ and $g(x)$ are piecewise continuous, bounded and absolutely integrable on the x -axis. Then prove that $\mathcal{F}(f^*g) = \sqrt{2\pi} \mathcal{F}(f) \mathcal{F}(g)$.29. If $\tilde{f}_n(k) = \mathcal{H}\{f(n)\}$, prove that

$$\mathcal{H}_n\left\{\left(\nabla^2 - \frac{n^2}{r^2}\right)f(r)\right\} = \mathcal{H}_n\left\{\frac{1}{r} \frac{d}{dr} \left(r \frac{df}{dr}\right) - \frac{n^2}{r^2} f(r)\right\} = -K^2 \tilde{f}_n(K)$$

provided both $r f'(r)$ and $r f(r)$ vanish as $r \rightarrow 0$ and $r \rightarrow \infty$.30. Solve : $u(n+2) - u(n+1) + u(n) = 0$, $u(n) = 0$, $u(1) = 2$.23. Prove that $\mathcal{M}\left(\frac{1}{e^x + e^{-x}}\right) = \Gamma(p)L(p)$ where $L(p) = \sum_{n=0}^{\infty} (-1)^n \frac{1}{(2n+1)^p}$.24. Find the inverse Z transform of $\frac{z^2}{(z-a)(z-b)}$.25. If $Z\{f(n)\} = F(z)$ and $Z\{g(n)\} = G(z)$, then prove that the Z transform of the convolution $f(n) * g(n)$ is given by $Z\{f(n) * g(n)\} = Z\{f(n)\}Z\{g(n)\}$, where

$$f(n) * g(n) = \sum_{m=0}^{\infty} f(n-m)g(m)$$

26. Using Z transform, find $\sum_{n=0}^{\infty} \frac{(-1)^n e^{-n}}{n+1}$.**SECTION – D**

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