Reg. No.:

Name :

V Semester B.Sc. Honours in Mathematics Degree (C.B.C.S.S. – O.B.E. –
Regular/Supplementary/Improvement) Examination, November 2024
(2021 and 2022 Admissions)

5B20 BMH: INTEGRAL TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

Time: 3 Hours

Max. Marks: 60

SECTION - A

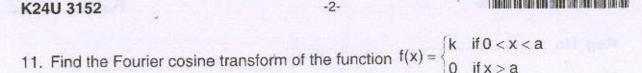
Answer any 4 questions out of 5 questions. Each question carries 1 mark. (4×1=4)

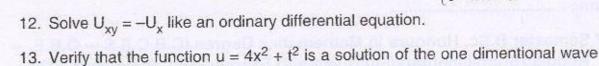
- Define Dirac delta function.
 Find the Landscape transform
- Find the Laplace transform of f(t) = t.
- Give an example of a non periodic function.
 Define inverse Fourier transform.
- 5. Write the two dimensional Laplace's equation.
- SECTION B

Answer any 6 questions out of 9 questions. Each question carries 2 marks. (6×2=12)

- Find the Laplace transform of the function f(t) = e^{at} where t ≥ 0, a is a constant.
- 7. Find $\mathcal{L}(e^{at} \sin \omega t)$.
- 8. Define a complex fourier series.
- Prove that the function (x² sin nx) is an even function.
 Define Fourier sine integral.

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- equation.

 14. Solve $U_{yy} = 0$ is an ordinary differential equation.
- SECTION C

Answer any 8 questions out of 12 questions. Each question carries 4 marks. (8x4=32)

15. Derive the formula L(cos ωt).16. State and prove the first shifting theorem.

- 17. Find $\mathcal{L}(t^{n+1})$.
- 18. Find the Fourier series of the function $f(x) = x + \pi$ if $-\pi < x < \pi$ and $f(x + 2\pi) = f(x)$.

20. State orthogonality of trigonometric system.

prove that $\mathcal{F}_{e}\{f'(x)\} = \omega \mathcal{F}_{s}\{f(x)\} - \sqrt{\left(\frac{2}{\pi}\right)}f(0)$.

- 18. Find the Fourier se
- 19. If f(x) and g(x) have period p, then show that h(x) = af(x) + bg(x), (a, b constants) has the period p.
- 21. Find the Fourier cosine integral of f(x) = e^{-kx}, where x > 0 and k > 0.
 22. Let f(x) be continuous and absolutely integrable on the x axis, let f'(x) be piecewise continuous on every finite interval and let f(x) → 0 as x → ∞. Then

23. Find the Fourier transform of $\mathcal{F}(e^{-ax})$ of $f(x) = \begin{cases} e^{-ax} & \text{if } x > 0 \\ 0 & \text{if } x < 0 \end{cases}$, Where a > 0.

24. Verify that $u = \frac{1}{\sqrt{x^2 + y^2 + z^2}}$ satisfies the Laplace's equation.

25. Find the D' Alembert's solution of the wave equation $U_{tt} - c^2 U_{xx} = 0$ with y = ct.

26. Solve $U_{xx} + 2U_{xy} + U_{yy} = 0$.

27. Solve the Volterra integral equation $y(t) - \int_0^t (1+\tau)y(t-\tau)d\tau = 1-\sinh t$.

28. Find the Fourier coefficients of the periodic function $f(x) = \begin{cases} -k & \text{if } -\pi < x < 0 \\ k & \text{if } 0 < x < \pi \end{cases}$ and $f(x + 2\pi) = f(x)$. Hence show that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + ... = \frac{\pi}{4}$.

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29. Find the Fourier transform of xe^{-x^2} using Fourier transform of derivatives.

30. Solve $f(x) = \begin{cases} U_0 = \text{constant} & \text{if } |x| < 1 \\ 0 & \text{if } |x| > 1 \end{cases}$ by the method of convolution.

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SECTION - D

Answer any 2 questions out of 4 questions. Each question carries 6 marks.