K24U 3152

Reg. No. : ...............................

Name : ...................................

**V Semester B.Sc. Honours in Mathematics Degree (C.B.C.S.S. – O.B.E. – Regular/Supplementary/Improvement) Examination, November 2024 (2021 and 2022 Admissions)**

**5B20 BMH : INTEGRAL TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS**

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any 4** questions out of 5 questions. **Each** question carries **1** mark. **(4×1=4)**

1. Define Dirac delta function.
2. Find the Laplace transform of f(t) = t.
3. Give an example of a non periodic function.
4. Define inverse Fourier transform.
5. Write the two dimensional Laplace's equation.

SECTION – B

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks. **(6×2=12)**

6. Find the Laplace transform of the function $f(t) = e^{at}$ where $t \geq 0$, a is a constant.
7. Find $\mathcal{L}(e^{at} \sin \omega t)$.
8. Define a complex fourier series.
9. Prove that the function $(x^2 \sin nx)$ is an even function.
10. Define Fourier sine integral.

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11. Find the Fourier cosine transform of the function $f(x) = \begin{cases} k & \text{if } 0 < x < a \\ 0 & \text{if } x > a \end{cases}$
12. Solve $U_{xy} = -U_x$ like an ordinary differential equation.
13. Verify that the function $u = 4x^2 + t^2$ is a solution of the one dimentional wave equation.
14. Solve $U_{yy} = 0$ is an ordinary differential equation.

SECTION – C

Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks. **(8×4=32)**

15. Derive the formula $\mathcal{L}(\cos \omega t)$.
16. State and prove the first shifting theorem.
17. Find $\mathcal{L}(t^{n+1})$.
18. Find the Fourier series of the function $f(x) = x + \pi$ if $-\pi < x < \pi$ and $f(x + 2\pi) = f(x)$.
19. If f(x) and g(x) have period p, then show that h(x) = af(x) + bg(x), (a, b constants) has the period p.
20. State orthogonality of trigonometric system.
21. Find the Fourier cosine integral of $f(x) = e^{-kx}$, where $x > 0$ and $k > 0$.
22. Let f(x) be continuous and absolutely integrable on the x – axis, let f′(x) be piecewise continuous on every finite interval and let $f(x) \to 0$ as $x \to \infty$. Then prove that $\mathcal{F}_e\{f'(x)\} = \omega \mathcal{F}_s\{f(x)\} - \sqrt{\left(\frac{2}{\pi}\right)} f(0)$.
23. Find the Fourier transform of $\mathcal{F}(e^{-ax})$ of $f(x) = \begin{cases} e^{-ax} & \text{if } x > 0 \\ 0 & \text{if } x < 0 \end{cases}$, Where $a > 0$.
24. Verify that $u = \frac{1}{\sqrt{x^2 + y^2 + z^2}}$ satisfies the Laplace's equation.
25. Find the D' Alembert's solution of the wave equation $U_{tt} - c^2 U_{xx} = 0$ with $y = ct$.
26. Solve $U_{xx} + 2U_{xy} + U_{yy} = 0$.



SECTION – D

Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks. **(2×6=12)**

27. Solve the Volterra integral equation $y(t) - \int_0^t (1+\tau) y(t-\tau) d\tau = 1 - \sinh t$.
28. Find the Fourier coefficients of the periodic function $f(x) = \begin{cases} -k & \text{if } -\pi < x < 0 \\ k & \text{if } 0 < x < \pi \end{cases}$ and $f(x + 2\pi) = f(x)$. Hence show that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots = \frac{\pi}{4}$.
29. Find the Fourier transform of $xe^{-x^2}$ using Fourier transform of derivatives.
30. Solve $f(x) = \begin{cases} U_0 = \text{constant} & \text{if } |x| < 1 \\ 0 & \text{if } |x| > 1 \end{cases}$ by the method of convolution.

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