Reg. No. : ....

Name : ....

V Semester B.Sc. Honours in Mathematics Degree (C.B.C.S.S. – Supplementary) Examination, November 2024
(2019 and 2020 Admissions)
BHM505 A : INTEGRAL EQUATIONS AND MEASURE THEORY

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any four** questions out of five questions. **Each** question carries **one** mark. **(4×1=4)**

1. Evaluate $\int_0^{\infty} \frac{e^{-nx}}{\sqrt{x}}\, dx$.
2. Find lim sup $\left(1-\frac{1}{n}\right)$.
3. Define counting measure.
4. Define the kernel of integral equation.
5. Find the Wronskian of $u(\xi) = \xi$, $v(\xi) = \frac{1}{\xi}$.

SECTION – B

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks. **(6×2=12)**

6. Let E = [0, 1]. Find $\chi_E(x)$.
7. Define step function and integral of step function.
8. If μ is a measure on X and A is a fixed set in X, then show that the function λ, defined by $E \in X$, by $\lambda(E) = \mu(A \cap E)$ is a measure on X.
9. Show that a measurable function f belongs to L if and only if |f| belongs to L.
10. If f is measurable, g is integrable and $|f| \le |g|$, then show that f is integrable and $\int |f|\, d\mu \le \int |g|\, d\mu$.

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11. Suppose that f; $|f| \in L$. Show that $|\int f\, d\mu| \le \int |f| d\mu$.
12. Show that constant multiple αf in L, that belongs to L, and $\int \alpha f\, d\mu = \alpha \int f\, d\mu$.
13. Define simple function and give an example.
14. Let $G(x, \xi) = \begin{cases} -\sin \xi \cos(x-1) & \text{if } 0 \le \xi \le x \\ -\sin x \cos(\xi - 1) & \text{if } x \le \xi \le 1 \end{cases}$. Show that G is symmetric.

SECTION – C

Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks. **(8×4=32)**

15. Define measurable space and measurable set.
16. Show that constant functions and characteristic functions are measurable.
17. If X is the set of real numbers, and **X** is the Borel algebra $\mathcal{B}$, then show that any continuous function f on R to R is Borel measurable. Also show that any monotone f on R to R is Borel measurable.
18. Let μ be a measure defined on a σ – algebra X. If (E_n) is an increasing sequence in X, then show that $\mu\left(\bigcup_{n=1}^{\infty} E_n\right) = \lim \mu(E_n)$.
19. Define Charge. Show that any finite linear combination of charges is charge.
20. Let λ denote Lebesgue measure defined on the Borel algebra **B** of R. If E is countable, then show that $E \in$ **B** and $\lambda(E) = 0$.
21. Suppose that f_1 and f_2 are in L(X, **X**, μ) and λ_1 and λ_2 be their indefinite integrals. Show that $\lambda_1(E) = \lambda_2(E)$, for all $E \in$ **X** if and only if $f_1(x) = f_2(x)$ for almost all x in X.
22. Find the kernel of the differential equation $\frac{d^2y}{dx^2} + \lambda y = 0$, $y(0) = 0$, $y(l) = 0$, by transforming into a integral equation.
23. Find the Green's function of the boundary value problem $\frac{d^2y}{dx^2} + y = f(x)$, $y(0) = 0$, $y'(1) = 0$.

24. Write down the four properties that have to be satisfied by Green's function of a second order differential equation with homogeneous boundary conditions.
25. Show that the characteristic numbers of a Fredholm equation with a real symmetric kernels are all real.
26. Find the resolvent Kernel of the integral equation $y(x) = 1 + \int_0^1 (1 - 3x\xi) y(\xi) d\xi$.

SECTION – D

Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks. **(2×6=12)**

27. Let f and g be measurable real-valued functions and let c be any real number. Then show that the functions cf, |f| and f g are measurable.
28. State and prove Fatous lemma.
29. State and prove Lebesgue dominated convergence theorem.
30. Obtain the most general solution of $y(x) = \lambda \int_0^1 (1 - 3x\xi) y(\xi) d\xi + F(x)$, where $F(x) \neq 0$, under the assumption $\lambda = 2, -2, \lambda \neq \pm 2$.

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