K24U 3153

Reg. No. : ....

Name : ....

V Semester B.Sc. Honours in Mathematics Degree (C.B.C.S.S. – O.B.E. – Regular/Supplementary/Improvement) Examination, November 2024
(2021 and 2022 Admissions)
5B21 BMH : ADVANCED LINEAR ALGEBRA

Time : 3 Hours | Max. Marks : 60

SECTION – A

Answer **any 4** questions out of 5 questions. **Each** question carries **1** mark. **(4×1=4)**

1. Define eigen vector of a square matrix.
2. Find the characteristic polynomial of $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$.
3. State generalised Pythagoras theorem.
4. Define complex inner product space.
5. Define unitarily diagonalisable matrix.

SECTION – B

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks. **(6×2=12)**

6. Find the eigen values of $A = \begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix}$.
7. Prove that similar matrices have the same characteristic polynomial.
8. Prove that, for $x, y \in R^2$, $<x, y> = x_1y_1 + 2x_2y_2$ is an inner product on R^2.
9. Prove that, for any $a, b \in R$ and any x, y, z in an inner product space V over R, $< z, ax + by > = a < z, x > + b < z, y >$.
10. Show that if P is an orthogonal matrix, then $|P| = \pm 1$.
11. True or False : Any projection is idempotent. Justify your answer.

P.T.O.

12. Let $A = \begin{bmatrix} 5 & -8 & -4 \\ 3 & -5 & -3 \\ -1 & 2 & 2 \end{bmatrix}$. Show that A is idempotent.
13. Show that the matrix $\begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix}$ is normal, but it is not hermitian.
14. Calculate the norm of the vector $x = (1, 0, i)^T \in C^3$.

SECTION – C

Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks. **(8×4=32)**

15. Find the eigen values of the matrix $A = \begin{bmatrix} -2 & 1 & -2 \\ -1 & 0 & 1 \\ 2 & 1 & 2 \end{bmatrix}$ and show that the matrix cannot be diagonalised over the real numbers.
16. Prove that 0 is an eigen value of A if and only if $AX = 0$ has a non-trivial solution.
17. Let $A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$. Find an invertible matrix P such that $P^{-1}AP$ is diagonal.
18. State and prove triangle inequality for norms.
19. Check whether the matrix $A = \begin{bmatrix} 7 & -15 \\ 2 & -4 \end{bmatrix}$ is orthogonally diagonalised or not.
20. Show that the vectors $v_1 = (1, 1)^T$ and $v_2 = (-1, 1)^T$ are eigen vectors of the symmetric matrix $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$ where $a, b \in R$. What are the corresponding eigen values ?
21. Prove that the following quadratic form $q(x, y, z) = 6xy - 4yz + 2xz - 4x^2 - 2y^2 - 4z^2$ is neither positive definite nor negative definite. Is it indefinite ?
22. Prove that orthogonal complement is a subspace of a vector space.
23. Show that the only eigen values of an idempotent matrix A are 0 and 1.

24. If A is a hermitian matrix, then prove that the eigen values of A are real.
25. If A is unitarily diagonalisable, then prove that A is normal.
26. Let $Y = \text{Lin}\{(1, 3, -1, 1)^T, (1, 4, 0, 2)^T\} \subset R^4$. Find a basis of $Y^{\perp}$.

SECTION – D

Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks. **(2×6=12)**

27. Consider the matrix A and the vector v : $A = \begin{bmatrix} -5 & 8 & 32 \\ 2 & 1 & -8 \\ -2 & 2 & 11 \end{bmatrix}, V = \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$.
    a) Show that v is an eigen vector of A and find the corresponding eigen value.
    b) Find all the eigen vectors corresponding to this eigen value and hence describe geometrically the eigen space.
28. Orthogonally diagonalise the matrix $A = \begin{bmatrix} 1 & -4 & 2 \\ -4 & 1 & -2 \\ 2 & -2 & -2 \end{bmatrix}$.
29. Prove that a linear transformation is a projection if and only if it is an idempotent.
30. Find the spectral decomposition of the matrix $A = \begin{bmatrix} 1 & i & 0 \\ i & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.
