K24U 0884

Reg. No. : ....

Name : ....

IV Semester B.Sc. Honours in Mathematics Degree (C.B.C.S.S. – O.B.E. – Regular/Supplementary/Improvement) Examination, April 2024
(2021 and 2022 Admissions)
4B17 BMH : ADVANCED STATISTICAL TECHNIQUES – II

Time : 3 Hours Max. Marks : 60

PART – A

Answer **any 4** questions out of 5 questions. **Each** question carries **one** mark.

1. What is the cumulant generating function of χ^2 distribution with n degrees of freedom ?
2. Define Fisher's 't'.
3. Define consistency of an estimator.
4. What do you mean by most efficient estimator ?
5. Define likelihood function. **(4×1=4)**

PART – B

Answer **any 6** questions out of 9 questions. **Each** question carries **two** marks.

6. Derive the moment generating function of χ^2 distribution with n degrees of freedom.
7. A machinist is making engine parts with axle diameters of 0.700 inch. A random sample of 10 parts shows a mean diameter of 0.742 inch with a standard deviation of 0.040 inch. Compute the statistic you would use to test whether the work is meeting the specifications.
8. What are the assumptions of t-test for difference of means ?
9. Define F-statistic and mention any one application of it.

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10. Mention the characteristics of a good estimator.
11. If $x_1, x_2, \ldots, x_n$ is a random sample from a normal population $N(\mu, 1)$, then show that $t = \frac{1}{n}\sum_{i=1}^{n} x_i^2$ is an unbiased estimator of $\mu^2 + 1$.
12. Prove that in sampling from a $N(\mu, \sigma^2)$ population, the sample mean is a consistent estimator of μ.
13. Prove that the maximum likelihood estimate of the parameter α of a population having density function $\frac{2}{\alpha^2}(\alpha - x)$, $0 < x < \alpha$ for a sample of unit size is 2x, x being the sample value.
14. Show that the sample mean $\bar{x}$ is sufficient for estimating the parameter λ of the Poisson distribution. **(6×2=12)**

PART – C

Answer **any 8** questions out of 12 questions. **Each** question carries **four** marks.

15. Mention the applications of Chi-square distribution.
16. The following table gives the number of aircraft accidents that occurs during the various days of the week. Find whether the accidents are uniformly distributed over the week.

Days	Sun.	Mon.	Tue.	Wed.	Thu.	Fri.	Sat.
No. of accidents	14	16	8	12	11	9	14

(Given : the values of chi-square significant at 5, 6, 7 d.f. are respectively 11.07, 12.59, 14.07 at the 5% level of significance)

17. For a 2 x 2 table,

a	b
c	d

Prove that chi-square test of independence gives $\chi^2 = \frac{N(ad - bc)^2}{(a+c)(b+d)(a+b)(c+d)}$, where N = a + b + c + d.

18. A random sample of 10 boys had the following I.Q.'s :
70, 120, 110, 101, 88, 83, 95, 98, 107, 100. Do these data support the assumption of a population mean I.Q. of 100 ? Find a reasonable range in which most of the mean I. Q. values of samples of 10 boys lie.
19. In one sample of 8 observations, the sum of the squares of deviations of the sample values from the sample mean was 84.4 and in the other sample of 10 observations it was 102.6. Test whether this difference is significant at 5 per cent level, given that the 5 per cent point of F for 7 and 9 d.f. is 3.29.
20. If T_1 and T_2 are two unbiased estimators of $\gamma(\theta)$, having the same variance and ρ is the correlation between them, then show that $\rho \geq 2e - 1$, where e is the efficiency of each estimator.
21. If T_n is a consistent estimator of $\gamma(\theta)$ and $\psi(\gamma(\theta))$ is a continuous function of $\gamma(\theta)$, then prove that $\psi(T_n)$ is a consistent estimator of $\psi(\gamma(\theta))$.
22. X_1, X_2 and X_3 is a random sample of size 3 from a population with mean value μ and variance σ^2 .T_1, T_2 and T_3 are the estimators used to estimate mean value μ where $T_1 = X_1 + X_2 - X_3$, $T_2 = 2X_1 + 3X_3 - 4X_2$, $T_3 = (\lambda X_1 + X_2 + X_3)/3$.
    i) Find the value of λ such that T_3 is unbiased estimator for μ.
    ii) Which is the best estimator ?
23. If T_1 is an MVUE of $\gamma(\theta)$, $\theta \in \Theta$ and T_2 is any other unbiased estimator of $\gamma(\theta)$ with efficiency e < 1, then prove that no unbiased linear combination of T_1 and T_2 can be an MVUE of $\gamma(\theta)$.
24. Prove that if a sufficient estimator exists, then it is a function of the maximum likelihood estimator.
25. Let $x_1, x_2, \ldots, x_n$ denote random sample of size n from a uniform population with pdf $f(x,\theta) = 1;\ \theta - \frac{1}{2} \leq x \leq \theta + \frac{1}{2},\ -\infty < \theta < \infty$. Obtain M.L.E. for θ.
26. In random sampling from normal population $N(\mu, \sigma^2)$, find the maximum likelihood estimator for μ when σ^2 is unknown. **(8×4=32)**

PART – D

Answer **any 2** questions out of 4 questions. **Each** question carries **six** marks.

27. It is believed that the precision (as measured by the variance) of an instrument is no more than 0.16. Write down the null and alternative hypothesis for testing this belief. Carry out the test at 1% level, given 11 measurements of the same subject on the instrument :
2.5, 2.3, 2.4, 2.3, 2.5, 2.7, 2.5, 2.6, 2.6, 2.7, 2.5.
28. A random sample of 16 values from a normal population showed a mean of 41.5 inches and the sum of squares of deviations from this mean equal to 135 square inches. Show that the assumption of a mean of 43.5 inches for the population is not reasonable. Obtain 95 per cent and 99 per cent fiducia limits for the same.
(Given $t_{0.05}$ for 15 d.f. = 2.131 and $t_{0.01}$ for 15 d.f. = 2.947)
29. Let $\{T_n\}$ be a sequence of estimators such that for all $\theta \in \Theta$,
    i) $E_\theta(T_n) \to \gamma(\theta)$, $n \to \infty$
    ii) $Var_\theta(T_n) \to 0$, $n \to \infty$
    Then prove that T_n is a consistent estimator of $\gamma(\theta)$.
30. Given one observation from a population with pdf $f(x, \theta) = \frac{2}{\theta^2}(\theta - x)$, $0 \leq x \leq \theta$.
Obtain $100(1 - \alpha)\%$ confidence interval for θ. **(2×6=12)**

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