K24U 0888

Reg. No. : ....

Name : ....

IV Semester B.Sc. Honours in Mathematics Degree (CBCSS – Supplementary/One Time Mercy Chance) Examination, April 2024
(2016 to 2020 Admissions)
BHM 403 : COMPLEX ANALYSIS, FOURIER SERIES AND PARTIAL DIFFERENTIAL EQUATIONS

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any 4** questions out of 5 questions. **Each** question carries **1** mark. **(4×1=4)**

1. Give an example of a function that is neither even nor odd.
2. Define accumulation point.
3. Give an example of an unbounded set.
4. Define order of the PDE.
5. Find the fundamental period of $\cos 2\pi x$.

SECTION – B

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks. **(6×2=12)**

6. Find the principal value of i^i.
7. Let $f(z) = \bar{z}$. Does $f'(z)$ exist at any point in complex plane ? Justify your answer.
8. Find the value of z for which $\sinh z = 0$.
9. Show that $\text{Log}(-ei) = 1 - \frac{\pi}{2}i$.
10. Solve for u given that $u_{yy} = 0$.

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11. Write the Fourier series of an even function of period 2L.
12. If f(x) has a period p then prove that the period of $f\left(\frac{x}{b}\right), b \neq 0$ is bp.
13. Graph $f(x) = |\sin x|$ for $-\pi < x < \pi$.
14. If a function f(z) is continuous and non zero at a point then prove that $f(z) \neq 0$ throughout some neighbourhood of that point.

SECTION – C

Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks. **(8×4=32)**

15. Show that $\lim_{z \to \infty} f(z) = \infty$ iff $\lim_{z \to 0} \frac{1}{f\left(\frac{1}{z}\right)} = 0$.
16. If f(z) is analytic in domain D and its conjugate $\overline{f(z)}$ is analytic in D, then prove that f(z) must be a constant in D.
17. Define harmonic function and check whether the function $u(x, y) = x^2 + y^2$ is harmonic. Justify.
18. Find the harmonic conjugate of the function $u(x, y) = 2x(1 - y)$.
19. Check whether the function $f(z) = (3x + y) + i(3y - x)$ is entire or not.
20. Find the image of the vertical and horizontal segments, under the transformation $w = e^z$.
21. Find all the roots of the equation $\sin z = \cosh 4$.
22. Prove the following.
    a) $\int_{-\pi}^{\pi} \cos nx \cos mx \, dx = 0 \ (n \neq m)$. Here n and m are integers.
    b) $\int_{-\pi}^{\pi} \sin nx \sin mx \, dx = 0 \ (n \neq m)$. Here n and m are integers.
    c) $\int_{-\pi}^{\pi} \sin nx \cos mx \, dx = 0 \ (n \neq m \text{ or } n = m)$. Here n and m are integers.
23. Find the Fourier series of the function $f(x) = \begin{cases} 0 & \text{if } -2 < x < -1 \\ k & \text{if } -1 < x < 1 \\ 0 & \text{if } 1 < x < 2 \end{cases}$, $p = 2L = 4$.

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24. Solve the PDE $u_{xy} = -u_x$.
25. Find a two dimensional Poisson equation whose solution is $u = \frac{1}{\sqrt{x^2 + y^2}}$.
26. Solve the system of PDEs $u_{xx} = 0$, $u_{yy} = 0$.

SECTION – D

Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks. **(2×6=12)**

27. Explain different steps involved in solving one dimensional wave equation $u_{tt} = c^2 u_{xx}$, $c^2 = \frac{T}{\rho}$ along with the boundary conditions $u(0, t) = 0$ and $u(L, t) = 0$, $\forall t \geq 0$ and initial conditions $u(x, 0) = f(x)$, $u_t(x, 0) = g(x)$ $(0 \leq x \leq L)$.
28. Find the type and transform into normal form and solve $u_{xx} - 2u_{xy} + u_{yy} = 0$.
29. Suppose that $f(z) = u(x, y) + iv(x, y)$ and that $f'(z)$ exists at a point $z_0 = x_0 + iy_0$. Then prove that the first order derivatives of u and v exist at (x_0, y_0) and they satisfy the Cauchy Riemann equations and $f'(z) = u_x + iv_x$.
30. Suppose that $f(z) = u(x, y) + iv(x, y)$ and $z_0 = x_0 + iy_0$ and $w_0 = u_0 + iv_0$ then prove that $\lim_{z \to z_0} f(z) = w_0$ iff $\lim_{(x,y) \to (x_0, y_0)} u(x, y) = u_0$ and $\lim_{(x,y) \to (x_0, y_0)} v(x, y) = v_0$.

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