K24U 0882

Reg. No. : ....

Name : ....

IV Semester B.Sc. Honours in Mathematics Degree (CBCSS – OBE-Regular/Supplementary/Improvement) Examination, April 2024 (2021 and 2022 Admissions)
4B15 BMH : INTRODUCTION TO ABSTRACT ALGEBRA AND LINEAR ALGEBRA

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any 4** questions out of 5 questions. **Each** question carries **1** mark. **(4×1=4)**

1. Define binary operation.
2. Give an example of a finite group that is not cyclic.
3. Order of A_n is
4. Write the standard basis of $\mathbb{R}^n$.
5. Define inverse of a linear transformation.

SECTION – B

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks. **(6×2=12)**

6. Prove that every cyclic group is abelian.
7. Prove that in a group the identity element is unique.
8. Define subgroup of a group. Is the set $\mathbb{Z}^+$, a subgroup of the group of complex numbers under addition ? Justify your answer.
9. Let $\sigma, \tau \in S_5$, if $\sigma = (1, 4, 3, 5)$ and $\tau = (1, 3, 4, 2, 5)$ find $\sigma\tau$.
10. Find the number of elements in the set $\{\sigma \in S_4 | \sigma(3) = 3\}$.
11. Express the vector $w = (2, -5)^T$ in R^2 as a linear combination of the vectors $v_1 = (1, 2)^T$ and $v_2 = (1, -1)^T$.

P.T.O.

12. Define linear span.
13. If $S, T : V \to V$ are linear transformations, then prove that the composition $ST : V \to V$ is a linear transformation.
14. What is the matrix of the linear transformation which is a reflection in the line $y = x$?

SECTION – C

Answer **any 8** questions out of 12 questions. **Each** question carries 4 marks. **(8×4=32)**

15. Check whether the set of all invertible $n \times n$ matrices under matrix multiplication is a group.
16. If H and K are subgroups of G, then prove that $H \cap K$ is a subgroup of G.
17. Find all non-trivial subgroups of $\mathbb{Z}_{12}$.
18. Prove that every group is isomorphic to a group of permutations.
19. Let σ be a permutation of set A, for $a, b \in A$, let $a \sim b$ if and only if $b = \sigma^n(a)$ for some $n \in \mathbb{Z}$. Prove that $\sim$ is an equivalence relation.
20. Describe the group S_3 by its group table.
21. Find the basis of the null space of the matrix $A = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \\ 0 & -1 & -1 \end{pmatrix}$.
22. For any $m \times n$ matrix A, prove that R(A) is a subspace of $\mathbb{R}^m$.
23. Find the rank of the matrix $A = \begin{pmatrix} 1 & 2 & 1 & 3 & 0 \\ 0 & 1 & 1 & 1 & -1 \\ 1 & 3 & 2 & 0 & 1 \end{pmatrix}$.
24. Let V be a finite-dimensional vector space and let T be a linear transformation from V to a vector space W. Then prove that T is completely determined by what it does to a basis of V.

25. Find the null space and range of the linear transformation $S : \mathbb{R}^2 \to \mathbb{R}^4$ given by $S\begin{pmatrix} x \\ y \end{pmatrix} = (x + y, x, x - y, y)^T$.
26. Suppose we change standard basis in $\mathbb{R}^2$ by a rotation of the axes $\frac{\pi}{4}$ radians anticlockwise. What are the coordinates of a vector $x = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ with respect to this new basis, $B = \{v_1, v_2\}$?

SECTION – D

Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks. **(2×6=12)**

27. Let G be a cyclic group with n elements and generated by a. Let $b \in G$ and let $b = a^s$. Then prove that b generates a cyclic subgroup H of G containing $\frac{n}{d}$ elements, where d is the greatest common divisor of n and s.
28. If $n \geq 2$, prove that the collection of all even permutations of $\{1, 2, ..., n\}$ forms a subgroup of the symmetric group S_n.
29. a) Define basis of a vector space.

    b) Let V be a vector space with a basis $B = \{v_1, v_2, ... v_n\}$ of n vectors. Then prove that any set of (n + 1) vectors are linearly dependent.
30. Verify rank-nullity theorem for the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x+y+z \\ y+z \\ z \end{pmatrix}$ and find the inverse of this linear transformation.

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