K24U 3602

Reg. No. : ....

Name : ....

III Semester B.Sc. Honours in Mathematics Degree (C.B.C.S.S. – O.B.E. – Regular/Supplementary/Improvement) Examination, November 2024
(2021 to 2023 Admissions)
3B10 BMH : CALCULUS – III

Time : 3 Hours   Max. Marks : 60

SECTION – A

Answer **any 4** questions out of 5 questions. **Each** question carries **1** mark. **(4×1=4)**

1. Define $\iint_D f(x, y)dA$ if D is a type 1 region.
2. Write an expression for the area of a surface with equation $z = f(x, y), (x, y) \in D$.
3. Define gradient vector field.
4. State Green's Theorem.
5. Give an example of a non orientable surface.

SECTION – B

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks. **(6×2=12)**

6. Evaluate the iterated integral $\int_1^4 \int_0^2 (6x^2y - 2x)\,dydx$.
7. Evaluate $\iint_D x\, dA$, where $D = \{(x, y) | 0 \leq x \leq \pi, 0 \leq y \leq \sin x\}$.
8. Evaluate $\int_{-3}^{3} \int_0^{\sqrt{9-x^2}} \sin(x^2 + y^2)\,dy\,dx$ by converting to polar coordinates.
9. A lamina with constant density $\rho(x, y) = \rho$ occupies the rectangle $0 \leq x \leq b$, $0 \leq y \leq h$. Find the moment of inertia about the x-axis.
10. Evaluate the triple integral $\iiint_B xyz^2\, dV$, where B is the rectangular box given by $B = \{(x, y, z) | 0 \leq x \leq 1, -1 \leq y \leq 2, 0 \leq z \leq 3\}$.
11. Write the equation $x^2 + z^2 = 9$ in spherical coordinates.
12. Find the gradient vector field of $f(x, y) = \sqrt{x^2 + y^2}$.

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13. If $F(x, y, z) = (x + yz)\, i + (y + xz)\, j + (z + xy)\, k$, find curl F.
14. Find a parametric representation of the part of the cylinder $y^2 + z^2 = 16$ that lies between the planes $x = 0$ and $x = 5$.

SECTION – C

Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks. **(8×4=32)**

15. Find the average value of $f(x, y) = x^2y$ over the rectangle R has vertices (–1, 0), (–1, 5), (1, 5), (1, 0).
16. Evaluate $\iint_D x \cos y\, dA$, where D is bounded by $y = 0$, $y = x^2$, $x = 1$.
17. Use polar coordinates to find the volume of the solid above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = 1$.
18. Find the center of mass of the lamina that occupies the region D is the triangular region with vertices (0, 0), (2, 1), (0, 3) and has the density function $\rho(x, y) = x + y$.
19. Find the area of the part of the plane $2x + 5y + z = 10$ that lies inside the cylinder $x^2 + y^2 = 9$.
20. Use spherical coordinates to evaluate $\iiint_B (x^2 + y^2 + z^2)^2\, dV$, where B is the ball with center the origin and radius 5.
21. If $F(x, y) = xy\, i + 3y^2 j$. Evaluate the line integral $\int_C F.dr$, where C is given by the vector function $r(t) = 11t^4 i + t^3 j$, $0 \leq t \leq 1$.
22. Find the work done by the force field $F = 2y^{3/2}\, i + 3\, x\, \sqrt{y}\, j$ in moving an object from P(1, 1) to Q(2, 4).
23. If $F(x, y) = i + \sin z\, j + y \cos z\, k$, find a function f such that $F = \nabla f$.
24. Show that the vector field $F(x, y, z) = xz\, i + xyz\, j - y^2 k$ can't be written as the curl of another vector field, that is, $F \neq \text{curl } G$.
25. Find the area of the part of the paraboloid $x = y^2 + z^2$ that lies inside the cylinder $y^2 + z^2 = 9$.
26. Evaluate $\iint_S z\, dS$, where S is the surface $x = y + 2z^2$, $0 \leq y \leq 1$, $0 \leq z \leq 1$.

SECTION – D

Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks. **(2×6=12)**

27. Find the volume of the solid under the plane $x - 2y + z = 1$ and above the region bounded by $x + y = 1$ and $x^2 + y = 1$.
28. Use the transformation $x = u^2 - v^2$, $y = 2uv$ to evaluate the integral $\iint_R y\, dA$, where R is the region bounded by the x-axis and the parabolas $y^2 = 4 - 4x$ and $y^2 = 4 + 4x$, $y \geq 0$.
29. Use Green's theorem to evaluate the line integral $\int_C y^3 dx - x^3 dy$, where C is the circle $x^2 + y^2 = 4$.
30. Use Stoke's Theorem to compute the integral $\iint_S \text{curl } F.\, dS$ where $F(x, y, z) = 2y \cos z\, i + e^x \sin z\, j + xe^y k$ and S is the hemisphere $x^2 + y^2 + z^2 = 9$, $z \geq 0$, oriented upward.