K24U 3607

Reg. No. : ....

Name : ....

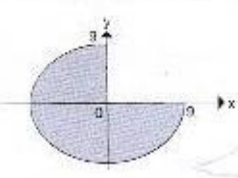
III Semester B.Sc. Honours in Mathematics Degree (C.B.C.S.S. – Supplementary) Examination, November 2024
(2019 and 2020 Admissions)
BHM302 : VECTOR CALCULUS

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any four** questions out of five questions. **Each** question carries **one** mark. **(4×1=4)**

1. Write the formula for the length of a smooth curve $r(t) = x(t)i + y(t)j + z(t)k$, $a \leq t \leq b$.
2. Find the gradient of the function $f(x, y) = \ln(x^2 + y^2)$ at the given point (1,1).
3. Describe the given region in polar coordinates.



4. Draw the graph of vector equation $r(t) = ti + (1 - t)j$, $0 \leq t \leq 1$.
5. Find the Curl of $F = x^2 + y^2 + z^2 + 1$.

SECTION – B

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks. **(6×2=12)**

6. Find the curve $r(t) = (2 \cos t)i + (2 \sin t)j + 25tk$, $0 \leq t \leq \pi$, unit tangent vector.
7. Let $f(x,y) = x - y$, $g(x, y) = 3y$. Find $\nabla(fg)$.

P.T.O.

8. Find the linearization $L(x, y)$ of the function $f(x,y) = (x + y + 2)^2$ at point (0, 0).
9. Evaluate the iterated integral $\int_1^2 \int_0^4 2xy \, dx \, dy$.
10. Find a spherical coordinate equation for the cone $z = \sqrt{x^2 + y^2}$.
11. Find the Jacobian of the transformation $x = \frac{u}{v}$, $y = uv$.
12. Integrate $f(x, y, z) = x - 3y^2 + z$ over the line segment C joining the origin to the point (1, 1, 1).
13. Find a parametrization of the sphere $x^2 + y^2 + z^2 = a^2$.
14. Integrate $G(x, y, z) = x^2$ over the cone $z = \sqrt{x^2 + y^2}$, $0 \leq z \leq 1$.

SECTION – C

Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks. **(8×4=32)**

15. Let $r(t) = (t + 1)i + (t2 - 1)j + 2tk$ is the position of a particle in space at time t. Find the particle's velocity and acceleration vectors. Then find the particle's speed and direction of motion at the given value of $t = 1$. Write the particle's velocity at that time as the product of its speed and direction.
16. Find the binormal vector and torsion for the space curve $r(t) = (3 \sin t)i + (3 \cos t)j + 4tk$.
17. Using the definition, find the derivative of $f(x, y) = x^2 + xy$ at (1, 2) in the direction of the unit vector $\frac{1}{\sqrt{2}}i + \frac{1}{\sqrt{2}}j$.
18. Let $f(x, y) = x^2 - xy + y^2 - y$. Find the directions u and the values of $D_u f(1, -1)$ for which
    a) $D_u f(1, -1)$ is largest
    b) $D_u f(1, -1)$ is smallest
    c) $D_u f(1, -1) = 0$
    d) $D_u f(1, -1) = 4$.
19. Find the volume of the region bounded above by the surface $z = 2 \sin x \cos y$ and below by the rectangle $R : 0 \leq x \leq \frac{\pi}{2}, 0 \leq y \leq \frac{\pi}{4}$.

20. Find the area of the region cut from the first quadrant by the curve $r = 2(2 - \sin 2\theta)^{\frac{1}{2}}$.
21. Find the average value of $F(x, y, z) = x^2 + 9$ over the cube in the first octant bounded by the coordinate planes and the planes $x = 2$, $y = 2$ and $z = 2$.
22. Find the circulation of the field $F = (x - y)i + xj$ around the circle $r(t) = (\cos t)i + (\sin t)j$, $0 \leq t \leq 2\pi$.
23. Find the work done by $F = (x^2 + y)i + (y^2 + x)j + ze^z k$ over the path the line segment $x = 1$, $y = 0$, $0 \leq z \leq 1$ from (1, 0, 0) to (1, 0, 1).
24. Find the surface area of a sphere of radius 2.
25. Find the flux of $F = yzj + z^2k$ outward through the surface S cut from the cylinder $y^2 + z^2 = 1$, $z \geq 0$, by the planes $x = 0$ and $x = 1$.
26. Find the circulation of the field $F = (x^2 - y)i + 4zj + x^2 k$ around the curve C in which the plane $z = 2$ meets the cone $z = \sqrt{x^2 + y^2}$, counterclockwise as viewed from above.

SECTION – D

Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks. **(2×6=12)**

27. Find unit tangent vector, principal unit normal vector, and curvature for the plane curve in $r(t) = ti + (\ln \cos t)j$, $-\pi/2 < t < \pi/2$.
28. Convert the integral $\int_{-1}^{1} \int_0^{\sqrt{1-y^2}} \int_0^{x} (x^2 + y^2) \, dz \, dx \, dy$, to an equivalent integral in cylindrical coordinates and evaluate the result.
29. Show that $F = (e^x \cos y + yz)i + (xz - e^x \sin y)j + (xy + z)k$ is conservative over its natural domain and find a potential function for it.
30. Let S be the "football" surface formed by rotating the curve $x = \cos z$, $y = 0$, $-\frac{\pi}{2} \leq z \leq \frac{\pi}{2}$ around the z-axis, Find a parametrization for S and compute its surface area.
