Reg. No. : ....

Name : ....

K24U 3606

III Semester B.Sc. Honours in Mathematics Degree (C.B.C.S.S. – Supplementary) Examination, November 2024
(2019 and 2020 Admissions)
BHM 301 : REAL ANALYSIS

Time : 3 Hours

Max. Marks : 60

PART – A

Answer **any 4** questions out of 5 questions. **Each** question carries **1** mark. **(4×1=4)**

1. State the Trichotomy Property.
2. When can you say that a sequence is convergent ?
3. Define an increasing sequence.
4. State Lipschitz condition.
5. Define conditionally convergent series.

PART – B

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks. **(6×2=12)**

6. If u and b ≠ 0 are elements in R with u.b = b, then prove that u =1.
7. State and prove the Triangle Inequality.
8. Prove that the sequence $(1 + (-1)^n)$ is not a Cauchy sequence.
9. Prove that a sequence in R can have at most one limit.
10. State and prove the n^{th} term test for convergence of a series.
11. If a series in R is absolutely convergent, then prove that it is convergent.

P.T.O.

12. Let $X := (x_n)$ be a nonzero sequence in R and let $a = \lim\left(n\left(1 - \left|\frac{x_n + 1}{x_n}\right|\right)\right)$ whenever this limit exists. Then prove that $\sum x_n$ is absolutely convergent when a > 1 and is no absolutely convergent when a < 1.
13. Let I = [a, b] be a closed, bounded interval and let f : I→R be continuous on I. If k ∈ R is any number satisfying in f f(I) ≤ k ≤ sup f(I) then prove that there exists a number c ∈ I such that f(c) = k.
14. If f : A→R is uniformly continuous on a subset A of R and if (x_n) is a Cauchy sequence in A, then prove that $(f(x_n))$ is a Cauchy sequence in R.

PART – C

Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks. **(8×4=32)**

15. Determine the set $B = \{x \in R : |x - 1| < |x|\}$.
16. State and prove the Archimedean Property.
17. If a and b are positive real numbers, then prove that their arithmetic mean is $\frac{a+b}{2}$, their geometric mean is $\sqrt{ab}$ and the Arithmetic-Geometric Mean inequality for a, b is $\sqrt{ab} \le \frac{a+b}{2}$ with equality occuring if and only if a = b.
18. State and prove Bolzanos Intermediate Value Theorem.
19. State and prove the Boundedness Theorem.
20. Let I be an interval and let f : I → R be continuous on I. Then prove that the set f (I) is an interval.
21. State and prove Monotone Subsequence Theorem.
22. Let $X = (x_n)$ be a sequence of real numbers that converges to x and suppose that $x_n \geq 0$. Then prove that the sequence $\sqrt{(x_n)}$ of positive square roots converges and $\lim \sqrt{(x_n)} = \sqrt{x}$.
23. Prove that every contractive sequence is a Cauchy sequence, and therefore is convergent.

24. State and prove the limit comparison test for convergence of a series.
25. Let $Z := (z_n)$ be a decreasing sequence of strictly positive numbers with $\lim(z_n) = 0$. Then prove that the alternating series $\sum(-1)^{n+1} z_n$ is convergent.
26. State and prove the integral test for convergence.

PART – D

Answer **any two** questions out of 4 questions. **Each** question carries **6** marks. **(2×6=12)**

27. Prove that there exists a positive real number x such that $x^2 = 2$.
28. State and prove the location of roots theorem.
29. Let $Y = (y_n)$ be defined inductively by $y_1 := 1$; $y_{n+1} := \frac{2y_n + 3}{4}$ for n ≥ 1. Find lim Y.
30. i) State and prove Abels lemma.

    ii) State and prove Abels test for convergence of series.

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