K24U 3601

Reg. No. : ....

Name : ....

III Semester B.Sc. Honours in Mathematics Degree (C.B.C.S.S. – O.B.E. – Regular/Supplementary/Improvement) Examination, November 2024
(2021 to 2023 Admissions)
3B09 BMH : REAL ANALYSIS

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any 4** questions. **Each** question carries **1** mark. **(4×1=4)**

1. Write distributive property of multiplication over addition.
2. State Trichotomy property.
3. Give an example of a set in which supremum does not belong to that set.
4. Define limit of a sequence.
5. State n^{th} term test for series.

SECTION – B

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks. **(6×2=12)**

6. Show that $1+2+...+n=\frac{n(n+1)}{2}$
7. Show that the set of even natural numbers is denumerable.
8. Prove that $|a + b| \le |a| + |b|$.
9. Use algebraic properties, show that $z + a = a \Rightarrow z = 0$.
10. List first 5 terms of the sequence inductively defined by $y_1 = 2, y_{n+1} = \frac{1}{2}(y_n + 1/y_n)$.
11. If $x_n \to x$ and $x_n \ge 0$, show that $x \ge 0$.

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12. Show that a convergent sequence is a Cauchy sequence.
13. Show that the harmonic series $\sum_{n=1}^{\infty}\frac{1}{n}$ diverges.
14. If $\sum_{n=1}^{\infty} a_n$ with $a_n > 0$ is convergent, then is $\sum_{n=1}^{\infty}\sqrt{a_n}$ always convergent ? Either prove it or give a counter example.

SECTION – C

Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks. **(8×4=32)**

15. Prove that $n^3 + 5n$ is divisible by 6 for all $n \in \mathbb{N}$.
16. Show that the set of rational numbers $\mathbb{Q}$ is denumerable.
17. Show that there does not exist a rational number r such that $r^2 = 2$.
18. State and prove Bernoulli's inequality.
19. Show that [0, 1] is uncountable.
20. Determine the set $B = \{x \in \mathbb{R} : x^2 + x > 2\}$.
21. Given that (x_n) is a sequence of positive real numbers such that $\frac{x_{n+1}}{x_n} \to L$. If $L < 1$, show that $x_n \to 0$.
22. Given that $a > 0$. Construct a sequence (s_n) of real numbers that converges to $\sqrt{a}$.
23. Determine the limit of the following sequences.
    a) $((3n)^{1/2n})$
    b) $((1 + 1/2n)^{3n})$.
24. State and prove comparison test for convergence of series.
25. Establish the convergence or divergence of the series $\sum_{n=1}^{\infty}\frac{\sin nx}{n}$.
26. a) Show that the series $\sum_{n=1}^{\infty}\cos n$ is divergent.
    b) Show that the series $\sum_{n=1}^{\infty}\frac{\cos n}{n^2}$ is convergent.

SECTION – D

Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks. **(2×6=12)**

27. State and prove Cantor's theorem.
28. State and prove characterisation theorem of intervals.
29. a) If $x_n \to x$ and $y_n \to y$, show that $x_n + y_n \to x + y$ and $x_n y_n \to xy$.
    b) Use an example, verify that the sum and the product of two divergent sequences may be convergent.
30. State ratio test. Discuss the convergence or the divergence of the series with n^{th} term :
    a) $\frac{n!}{e^{-n}}$
    b) $\frac{n!}{e^{-n^2}}$

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