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K24U 1804

Reg. No.: .....

Name : .....

Second Semester B.Sc. (Hon's) Mathematics Degree
(CBCSS – Supplementary/One Time Mercy Chance)
Examination, April 2024
(2016 – 2020 Admissions)
BHM 203 : INTEGRAL CALCULUS

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any 4** questions out of 5 questions. **Each** question carries **1** mark. **(4×1=4)**

1. Find a formula for the n^{th} term of the sequence 2, 6,10,14,18, ...
2. Given $a_1 = 1$, $a_{n+1} = \frac{a_n}{n+1}$. Find a_2, a_3, a_4, a_5.
3. Find the sum of the geometric series $\sum_{n=1}^{\infty} \frac{(-1)^n 5}{4^n}$.
4. Find $\frac{d}{dx}\int_{-\pi}^{x} \cos t\, dt$.
5. Define Taylor Series generated by a function f(x) at x = a.

SECTION – B

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks. **(6×2=12)**

6. For what values of x do the power series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots$ converge ?
7. Find the Taylor series and Taylor polynomials generated by f(x) = cos x at x = 0.

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8. Express the solution of the initial value problem $\frac{dy}{dx} = \sec x,\ y(0) = 4$.
9. Express the limit $\lim_{\|P\|\to 0} \sum_{k=1}^{n} \left(\frac{1}{c_k}\right)\Delta x_k$, where P is a partition of [1, 4].
10. Evaluate $\int_{-1}^{1} 3x^2\sqrt{x^3+1}\, dx$.
11. Find the area between y = 2 sin x and y = sin 2x, $0 \le x \le \pi$.
12. Find the volume of the solid generated by revolving the region between the y – axis and the curve $x = \frac{2}{y}$, $1 \le y \le 4$, about the y – axis.
13. Using reduction formula, evaluate $\int \sin^5 x\, dx$.
14. Using reduction formula, evaluate $\int \tan^4 x\, dx$.

SECTION – C

Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks. **(8×4=32)**

15. Find the Maclaurin series for cos 2x.
16. Find the sum of the series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$.
17. Find the Taylor series expansion about x = 0 of $\frac{1}{3}(2x + x\cos x)$.
18. Find the length of the curve $y = \left(\frac{x}{2}\right)^{\frac{2}{3}}$ from x = 0 to x = 2.
19. Show that the center of mass of a straight, thin strip or rod of constant density lies halfway between its two ends.
20. Define work done by a force F(x) along x – axis. Calculate the work done by $F(x) = \frac{1}{x^2}$ N along the x – axis from x = 1m to x = 10 m.
21. It took 1800 J of work to stretch a spring from its natural length of 2 m to a length of 5 m. Find the Spring's force constant.

22. Evaluate $\int_0^1 \frac{x^3}{\sqrt{x^4+9}}$ and $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(2 + \tan\frac{t}{2}\right)\sec^2\frac{t}{2}\, dt$.
23. Using the definition of limits, show that $\lim_{n\to\infty} k = k$.
24. State the n^{th} term test for divergence and hence test the divergence of the series $\sum_{n=1}^{\infty} \frac{n+1}{n}$.
25. State second part of the Fundamental Theorem of Calculus and evaluate $\int_{-\pi/4}^{0} \sec x \tan x\, dx$.
26. At what point in the interval $\left[0, \sqrt{3}\right]$, does the function $f(x) = x^2 - 1$ assumes its average value ?

SECTION – D

Answer **any 2** questions out of 4 questions. **Each** carries **6** marks. **(2×6=12)**

27. Find the area inside the smaller loop of the limacon $r = 2\cos\theta + 1$.
28. Find the length of the cardioid $r = 1 - \cos\theta$.
29. Give the reduction formula for $\int_0^{\frac{\pi}{2}} \sin^n x\, dx$ and evaluate $\int_0^1 \frac{x^n}{\sqrt{1-x^2}} dx$.
30. a) Use the Max-Min Inequality to find the upper and lower bounds for the value of $\int_0^1 \frac{1}{1+x^2} dx$.

    b) Use the inequality $\sin x \le x$, which holds for $x \ge 0$, to find an upper bound for the value of $\int_0^1 \sin x\, dx$.