K24U 1803

Reg. No. : ....

Name : ....

Second Semester B.Sc. Hon's (Mathematics) Degree (CBCSS – Supplementary/One Time Mercy Chance) Examination, April 2024
(2016 – 2020 Admissions)
BHM 202 : ABSTRACT ALGEBRA AND LINEAR ALGEBRA

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any 4** questions out of 5 questions. **Each** question carries **1** mark.

1. Give an example of a binary operation on the set $\mathbb{Z}$.
2. Write an example of a non-abelian group.
3. Give an example of a linearly independent set in $\mathbb{R}^2$ over $\mathbb{R}$.
4. What is the dimension of $\mathbb{R}^3$ over $\mathbb{R}$?
5. Is $T : \mathbb{R} \to \mathbb{R}$ defined by $T(x) = x + 1$ linear ? Justify your answer. **(4×1=4)**

SECTION – B

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks.

6. Show that in a group G with the binary operation $*$, $a * b = a * c$ implies $b = c$.
7. Write the additive inverse of every non zero elements in the group $\mathbb{Z}_6$.
8. Find the number of generators in the group $\mathbb{Z}_{23}$.
9. Define permutation groups.
10. Find (1, 4, 5, 6) (2, 1, 5).
11. Write all subspaces of $\mathbb{R}^2$ over $\mathbb{R}$.
12. Check whether the set $W = \{(x, y) : x + y = 1\}$ is subspace of $\mathbb{R}^2$.

P.T.O.

13. Show that the map $T : \mathbb{R} \to \mathbb{R}$, defined by $T(x) = ax$ is linear.
14. Let $T : \mathbb{R} \to \mathbb{R}$ defined by $T(x) = (x, x)$. Find range space and null space of T. **(6×2=12)**

SECTION – C

Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks.

15. Define the binary operation $*$ on $\mathbb{Z}$ by $x * y = x + y + 1$. Show that $(\mathbb{Z}, *)$ is an abelian group.
16. In a group G show that
    i) $(a^{-1})^{-1} = a$
    ii) $(ab)^{-1} = b^{-1} a^{-1}$
17. Find all subgroups of the group $\mathbb{Z}_5$. Give the group table of the group.
18. Show that in a group G, if $a \in G$ be a generator, then a^{-1} is also a generator.
19. Let $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 3 & 5 & 4 & 6 \end{pmatrix}$ and $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 1 & 2 & 4 & 3 & 5 \end{pmatrix}$. Compute β^{-1} and $\beta\alpha$.
20. Find the order of the following permutations.
    i) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 5 & 4 & 3 & 6 \end{pmatrix}$
    ii) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 7 & 6 & 1 & 2 & 3 & 4 & 5 \end{pmatrix}$
21. Find the dimension of the subspace $W = \{(x, y, z) : x + y - z = 0\}$.
22. Determine whether the polynomials $\{1 - x, 5 + 3x - 2x^2, 1 + 3x - x^2\}$, are linearly independent or not.
23. Write a basis for $\mathbb{R}^2$ over $\mathbb{R}$. Express (1, 0) as the linear combination of the basis.

24. Find the null space and the nullity of the linear transformation $T(x, y, z) = (x + y, x - y, z)$.
25. The linear map $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x, y) = (x + y, x)$. Show that T is non-singular and find T^{-1}.
26. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x, y) = (-y, x)$. Find matrix of T with respect to the standard basis. **(8×4=32)**

SECTION – D

Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks.

27. Show that a group G is abelian if and only if $(ab)^2 = a^2b^2$, $\forall$ a, b $\in$ G.
28. Show that subgroups of a cyclic group are cyclic.
29. Show that in a finite dimensional vector space V, any two bases have same number of elements.
30. Find the range and kernel of the linear transformation $T(x, y, z) = (x + z, x + y + 2z, 2x + y + 3z)$. **(2×6=12)**