Reg. No. : ....

Name : ....

II Semester B.Sc. Mathematics (Hon's) Degree (C.B.C.S.S. – OBE – Regular/ Supplementary/Improvement) Examination, April 2024
(2021 Admission Onwards)
Core Course
2B05BMH : CALCULUS – II

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any 4** questions. **Each** question carries **1** mark. **(4×1=4)**

1. Convert the point $\left(2, \frac{\pi}{3}\right)$ from polar to Cartesian Coordinates.
2. Write down the formula for finding the length of a curve with polar equation $r = f(\theta)$ from the point at which $\theta = a$ to the point where $\theta = b$.
3. What does it mean to say that $\lim_{n\to\infty} a_n = 8$?
4. State limit comparison test for series.
5. Write down the general equation of the hyperbolic paraboloid.

SECTION – B

Answer **any 6** out of 9 questions. **Each** question carries **2** marks. **(6×2=12)**

6. Show that the surface area of a sphere of radius r is $4\pi r^2$.
7. Find the slope of the tangent to the curve $x = 1 + \ln(t)$, $y = t^2 + 2$ at the point (1, 3).
8. Determine whether the sequence $a_n = \frac{\sin 2n}{1+\sqrt{n}}$ converge or diverge. If it converge, find the limit.
9. If $\lim_{n\to\infty} |a_n| = 0$, then show that $\lim_{n\to\infty} a_n = 0$.

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10. Check whether the series $\sum_{n=1}^{\infty} \frac{e^n}{\left(1+\frac{1}{n}\right)}$ converges or diverges. Justify your answer.
11. Show that if $|r(t)| = c$ (a constant), then $r'(t)$ is orthogonal to $r(t)$ for all t.
12. Find the equation of a plane through the point (6, 3, 2) and perpendicular to the vector $< -2, 1, 5 >$.
13. Find $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$, if $u = x^{\frac{y}{z}}$.
14. Find $D_u f(2, -1)$, if $f(x, y) = x^2y^3 - 4y$ and $u = \frac{2}{\sqrt{29}} i + \frac{5}{29} j$.

SECTION – C

Answer **any 8** out of 12 questions. **Each** question carries **4** marks. **(8×4=32)**

15. Find the surface area obtained by rotating the given curve about x-axis. $x = t^3$, $y = t^2$, $0 \le t \le 1$.
16. Prove that if $\lim_{n\to\infty} a_n = 0$ and $\{b_n\}$ is bounded, then $\lim_{n\to\infty} (a_n b_n) = 0$.
17. Determine whether the series $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$ converges or diverges. Justify your answer.
18. Determine whether the series $\sum_{n=1}^{\infty} \frac{(-1)^n e^{\frac{1}{n}}}{n^3}$ is absolutely convergent, conditionally convergent or divergent. Justify your answer.
19. Evaluate $\int e^{-x^2} dx$ as an infinite series.
20. Find symmetric equations for the line of intersection L of the planes $x + y + z = 1$ and $x - 2y + 3z = 1$.
21. Find the arc length of the circular helix with vector equation $r(\vec{t}) = \cos t i + \sin t j + t k$ from the point (1, 0, 0) to the point (1, 0, 2π).
22. If $z = e^x \sin y$, where $x = st^2$ and $y = s^2t$, find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$.

23. Find directional derivative of $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$ at (1, 2, –2) in the direction of $\vec{v} = -\frac{2}{3} i + \frac{2}{3} j - \frac{1}{3} k$.
24. A particle moves with position function $r(\vec{t}) = t^2 i + t^2 j + t^3 k$. Find the tangential and normal components of acceleration.
25. Determine the set of points at which the function.

$$f(x, y) = \begin{cases} \frac{x^2y^3}{2x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 1 & \text{if } (x, y) = (0, 0) \end{cases}$$ is continuous.

26. Let f and g are twice differentiable function of a single variable. Show that the function $u(x, t) = f(x + at) + g(x - at)$ is a solution of the wave equation $u_{tt} = a^2 u_{xx}$.

SECTION – D

Answer **any 2** out of 4 questions. **Each** question carries **6** marks. **(2×6=12)**

27. a) Let $r(\vec{t})$ be a vector valued function. Show that the curvature of the curve given by $r(\vec{t})$ is, $K(t) = \frac{|r'(t) \times r''(t)|}{|r'(t)|^3}$.
    b) Find the radius of convergence and interval of convergence of the series $\sum_{n=0}^{\infty} \frac{n(x+2)^n}{3^{n+1}}$.
28. a) Find the directional derivative of $f(x, y, z) = x^3 - xy^2 - z$ at $P_0(1, 1, 0)$ in the direction of $v = 2i - 3j + 6k$.
    b) Use Lagrange multipliers to prove that the rectangle with maximum area that has a given perimeter P is a square.

29. a) Find parametric equations for the tangent line to the curve $x = 1 + 2\sqrt{t}$, $y = t^3 - t$, $z = t^3 + t$ at the point (3, 0, 2).
    b) Find an equation of the plane that passes through the points (0, –2, 5) and (–1, 3, 1) and is perpendicular to the plane $2z = 5x + 4y$.
30. a) A rectangular box without a lid to be made from $12 m^2$ of cardboard. Find the maximum volume of such a box.
    b) Find the area of the region enclosed by one loop of the curve $r = 4 \cos 3\theta$.