K24U 1806

Reg. No. : ....

Name : ....

Second Semester B.Sc. Hon's (Mathematics) Degree (C.B.C.S.S. – Supplementary/One Time Mercy Chance) Examination, April 2024
(2016 – 2020 Admissions)
BHM 204 : THEORY OF NUMBERS AND EQUATIONS

Time : 3 Hours Max. Marks : 60

SECTION – A

Any 4 out of 5 questions. **Each** question carries **1** mark.

1. Define the greatest common divisor of a and b.
2. When will you say that two integers are relatively prime ?
3. For integers a, b, c if a|b and a|c then prove that a|bx + cy.
4. Define a complete set of residues modulo n.
5. Remove the fractional coefficients from the equation $x^3 - \frac{1}{4}x^2 + \frac{1}{2}x - 1 = 0$.

(4×1=4)

SECTION – B

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks.

6. If gcd(a, d) = d then prove that gcd(a/d, b/d) = 1.
7. For any integer $k \neq 0$, gcd(ka, kb) = |k|gcd(a, b).
8. If p is a prime and p|ab then prove that p|a or p|b.
9. Prove that $2^{20} \equiv 1 \pmod{41}$.
10. Find the remainder obtained upon dividing the sum 1! + 2! + 3! + ... +100! by 12.
11. Solve the linear Diophantine equation 9x – 30y = 21.

P.T.O.

12. Find the number of divisors and sum of divisors of 180.
13. Form a rational cubic equation which shall have for roots 1, $3-\sqrt{-2}$.
14. If f(x) = 0 is an equation of odd degree prove that it has at least one real root whose sign is opposite to that of last term.

(6×2=12)

SECTION – C

Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks.

15. For positive integers a and b prove that gcd(a, b)1cm(a, b) = ab.
16. Prove that $\sqrt{2}$ is irrational.
17. Prove that there are infinite number of primes.
18. Explain the Euclidean Algorithm and using it find the gcd(12378, 3054).
19. State and Prove Euclid's lemma.
20. Prove that the gcd of any two integers not both of which are zero can be expressed as the linear combination of a and b.
21. State and prove Fermat's little theorem.
22. Prove that in an equation with real coefficients the imaginary roots occur in pairs.
23. Determine completely the nature of the roots of the equation $x^5 - 6x^2 - 4x + 5 = 0$.
24. Find the multiple roots of the equation $x^4 - 9x^2 + 4x + 12 = 0$.
25. Find the number and position of real roots of the equation $x^3 - 3x + 6 = 0$.
26. If α, β, γ are the roots of the equation $x^3 + ax^2 + bx + c = 0$, form the equation whose roots are $\alpha\beta$, $\beta\gamma$ and $\gamma\alpha$.

(8×4=32)

SECTION – D

Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks.

27. State and prove Fundamental Theorem of Arithmetic.
28. State and prove Wilson's theorem.
29. Prove that every equation f(x) = 0 of the n^{th} degree has n roots and no more.
30. Find the roots of the equation $x^5 + 4x^4 + 3x^3 + 3x^2 + 4x + 1 = 0$.

(2×6=12)

____