K24U 1717

Reg. No. : ....

Name : ....

Second Semester B.Sc. (Hon's) Mathematics Degree (CBCSS – OBE – Regular/Supplementary/Improvement) Examination, April 2024
(2021 Admission Onwards)
Core Course
2B07 BMH : THEORY OF NUMBERS AND EQUATIONS

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any 4** questions from this Section. **Each** question carries **1** mark. **(4×1=4)**

1. State division algorithm.
2. What do you mean by the set of least non-negative residues modulo n ?
3. State Euler's theorem.
4. State remainder theorem.
5. If α, β are the roots of the equation $ax^2 + bx + c = 0$, what is the value of $\alpha + \beta$?

SECTION – B

Answer **any 6** questions from this Section. **Each** question carries **2** marks. **(6×2=12)**

6. State Euclidean algorithm.
7. If $k > 0$, then prove that $\gcd(ka, kb) = k \gcd(a, b)$.
8. Define a prime number. Give the first five prime numbers.
9. If $a \equiv b \pmod n$ and $b \equiv c \pmod n$, then prove that $a \equiv c \pmod n$.
10. Show that $18! \equiv -1 \pmod{437}$.
11. What do you mean by multiplicative function ? Give an example.

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12. Form a rational cubic equation whose two roots are 1 and $3 - i\sqrt{2}$.
13. If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$. express the value of $\Sigma\alpha^2\beta$ in terms of the coefficients.
14. Find the multiple roots of the equation $x^4 - 9x^2 + 4x + 12 = 0$.

SECTION – C

Answer **any 8** questions from this Section. **Each** question carries **4** marks. **(8×4=32)**

15. Show that the cube of any integer is of the form 7k or 7k ± 1.
16. Given integers a and b, not both of which are zero, prove that there exist integers x and y such that $\gcd(a, b) = ax + by$.
17. For positive integers a and b, prove that $\gcd(a, b) \times \operatorname{lcm}(a, b) = ab$.
18. Using Euclidean algorithm, find the gcd of 12378 and 3054. Express the gcd as a linear combination of 12378 and 3054.
19. If n is an odd pseudoprime, then prove that $M_n = 2^n - 1$ is a larger one.
20. Using Euler's theorem, prove that for any integer a, $a^{37} \equiv a \pmod{1729}$.
21. Solve the linear congruence equation $17x \equiv 9 \pmod{276}$.
22. If f is a multiplicative function and F is defined by $F(n) = \sum_{d|n} f(d)$, then prove that F is also multiplicative.
23. If α is a real root of the cubic equation $x^3 + px^2 + qx + r = 0$, the coefficients are reals, then show that the other two roots of the equations are real if $p^2 > 4q + 2p\alpha + 3\alpha^2$.
24. Show that $\frac{a^2}{x-\alpha} + \frac{b^2}{x-\beta} + \frac{c^2}{x-\gamma} - x + \delta = 0$ has only real roots if a, b, c, $\alpha, \beta, \gamma, \delta$ are all real.
25. Find the roots of the equation $x^5 + 4x^4 + 3x^3 + 3x^2 + 4x + 1 = 0$.
26. Find the values of 'a' for which $ax^3 - 9x^2 + 12x - 5 = 0$ has equal roots and solve in one case.

SECTION – D

Answer **any 2** questions from this Section. **Each** question carries **6** marks. **(2×6=12)**

27. Prove that the linear Diophantine equation $ax + by = c$ has a solution if and only if d|c, where $d = \gcd(a, b)$. If x_0, y_0 is any particular solution of this equation, then prove that all other solutions are given by

$$x = x_0 + \left(\frac{b}{d}\right)t,\ y = y_0 - \left(\frac{a}{d}\right)t$$

where t is an arbitrary integer.

28. State and prove Fermat's theorem.
29. Show that the roots of the equation $x^3 + px^2 + qx + r = 0$ are in arithmetical progression if $2p^3 - 9pq + 27r = 0$. Hence solve the equation $x^3 - 6x^2 + 13x - 10 = 0$.
30. Solve the equation $x^4 - 4x^3 - 10x^2 + 64x + 40 = 0$.