K24U 4132

Reg. No. : ....

Name : ....

First Semester B.Sc. Honours in Mathematics (C.B.C.S.S. – O.B.E. – Supplementary/Improvement) Examination, November 2024
(2021 to 2023 Admission)
Core Course
1B02BMH : FOUNDATIONS OF MATHEMATICS

Time : 3 Hours Max. Marks : 60

PART – A

Answer **any 4** questions. **Each** question carries **1** mark. **(4×1=4)**

1. Define Range of an $m \times n$ matrix A.
2. What is the graphical characterization of a bijective function defined on $\mathbb{R}$?
3. Define partition of a non-empty set.
4. Define hyperplane in $\mathbb{R}^n$.
5. Define Kernal of an $m \times n$ matrix A.

PART – B

Answer **any 6** questions. **Each** question carries **2** marks. **(6×2=12)**

6. $f : X \to Y$ such that $f(A \cap B) = f(A) \cap f(B)$ holds for all subsets A and B of X. Show that f is one-one.
7. $f : X \to Y$ and $g : Y \to Z$ be two one-one functions. Is $g_o f$ one-one ? Justify.
8. Let $X = \mathbb{N} \times \mathbb{N}$. Define $(m, n) \sim (p, q)$ iff $m + q = p + n$. Show that the relation is transitive.
9. Write example of a relation that is not a function.
10. Find the angle between vectors $\begin{pmatrix}1\\2\end{pmatrix}$ and $\begin{pmatrix}3\\1\end{pmatrix}$.
11. State basic properties of inner product of $\mathbb{R}^n$.
12. State elementary row operations.

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13. Write an example for augmented matrix.
14. When do we say that a matrix is in Row echelon form ?

PART – C

Answer **any 8** questions. **Each** question carries **4** marks. **(8×4=32)**

15. $f : [0, \pi) \to \mathbb{R}$ given by $f(x) = \cos x$. Check whether f is one-one.
16. $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = x^2$. Show that $f([-4, 3]) = [0, 16]$.
17. Show that a function $f : X \to Y$ is one-one iff $A = f^{-1}(f(A))$ for each $A \subseteq X$.
18. Define equivalence relation. Let X be a non-empty set and ~ be an equivalence relation on X. If $y \in [x]$, then show that $[x] = [y]$.
19. Write the equivalence classes and the transversal of the relation $R = \{(x, y) \in \mathbb{R} \times \mathbb{R} : xy > 0\} \cup \{(0, 0)\}$.
20. Let $X = Z^*$, set of non-zero integers. Define a relation R on X by mRn iff m divides n. Check whether the relation is reflexive, symmetric, anti-symmetric and transitive.
21. Show that the planes $x + 2y - 3z = 0$ and $x - 2y + 5z = 4$ intersect in a line. Find vector equation of line of intersection.
22. If $v = (1, 2)^T$, then find a unit vector in the same direction as v.
23. Write the Cartesian equation of the plane
$$\begin{pmatrix}x\\y\\z\end{pmatrix} = s\begin{pmatrix}1\\2\\-1\end{pmatrix} + t\begin{pmatrix}-2\\1\\7\end{pmatrix}; s, t \in \mathbb{R}.$$
24. Define rank of a matrix M. Find the rank of the matrix $M = \begin{pmatrix}1&2&1&1\\2&3&0&5\\3&5&1&6\end{pmatrix}$.
25. Solve the system of equations using Gaussian Elimination
$x + y + z = 3$
$2x + y + z = 4$
$x - y + 2z = 5$.
26. Check whether the system of equations are consistent or inconsistent
$x + y + 3z + w = 2, x - y + z + w = 4, y + 2z + 2w = 0$.

PART – D

Answer **any 2** questions. **Each** question carries **6** marks. **(2×6=12)**

27. State and prove the following :
    a) The Induction Principle ⇒ The Strong Induction Principle.
    b) The Well Ordering Principle ⇒ The Induction Principle.
28. State and prove Cantor's theorem.
29. If A is an $n \times n$ matrix, show that the following statements are equivalent.
    a) A^{-1} exist.
    b) $Ax = b$ has a unique solution for any $b \in \mathbb{R}^n$.
    c) $Ax = 0$ only has trivial solution $x = 0$.
    d) The reduced row echelon form of A is I.
30. Are the following lines L_1 and L_2 intersecting, parallel or skew ?
$$L_1 : \begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}1\\3\\4\end{pmatrix} + t\begin{pmatrix}1\\2\\-1\end{pmatrix}; t \in \mathbb{R}$$
$$L_2 : \begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}5\\6\\1\end{pmatrix} + t\begin{pmatrix}-2\\1\\7\end{pmatrix}; t \in \mathbb{R}.$$